# Re-basin via implicit Sinkhorn differentiation

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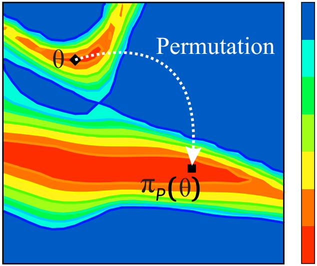
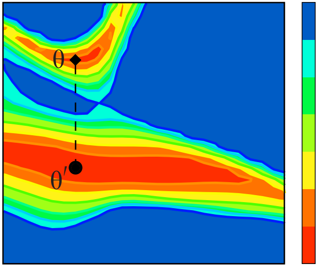
# Introduction

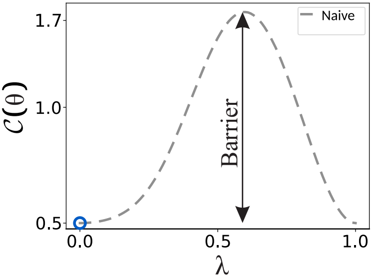
**Re-basin:** Find a suitable permutation of the parameters such that minimizes a given objective



# Introduction

**Example:** Re-basin for linear mode connectivity



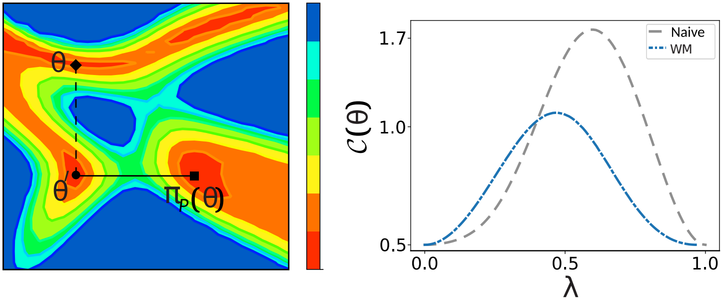




**Objective:** Find the permutation of $\theta$ that minimizes Barrier$(\theta, \theta')$

$$\text{Barrier}(\theta,\theta') = \sup_{\lambda} \left[ \left[ \mathcal{C}\left( (1-\lambda)\theta + \lambda\theta' \right) \right] - \left[ (1-\lambda)\mathcal{C}(\theta) + \lambda\mathcal{C}(\theta') \right] \right]$$

# Introduction

**Example:** Re-basin for linear mode connectivity




WM - Weight Matching re-basin [Ainsworth et al., 2022]

# Re-basin via implicit Sinkhorn differentiation

Learning the permutations using Sinkhorn operator

**Model:** $\ell_i'(z) = \sigma(S_\tau(P_i)W_i S_\tau(P_{i-1}^T)z + S_\tau(P_i)b_i)$

**Cost functions:**

L2 distance loss: $\mathcal{C}_{L2}(\mathcal{P};\theta_A,\theta_B) = ||\theta_A - \pi_{\mathcal{P}}(\theta_B)||^2$

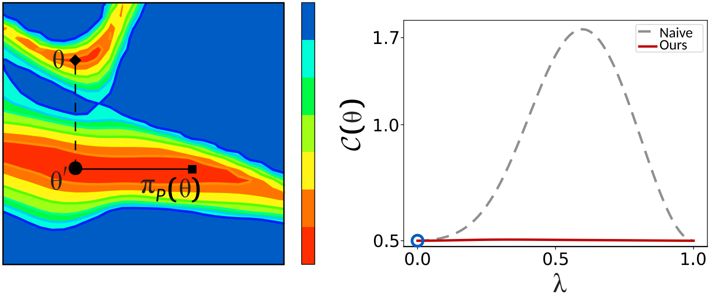
Middle point loss: $\mathcal{C}_{Mid}(\mathcal{P};\theta_A,\theta_B) = \mathcal{C}\left(\frac{\theta_A + \pi_{\mathcal{P}}(\theta_B)}{2}\right)$

Random point loss: $\mathcal{C}_{Rnd}(\mathcal{P};\theta_A,\theta_B) = \mathcal{C}\left((1-\lambda)\theta_A + \lambda\pi_{\mathcal{P}}(\theta_B)\right)$, $\lambda \sim U(0,1)$

**Optimization problem:** $\mathcal{P}^* = \arg\min_{\mathcal{P}} \mathcal{C}(\mathcal{P};\theta_A,\theta_B)$

# Introduction

**Example:** Re-basin for linear mode connectivity

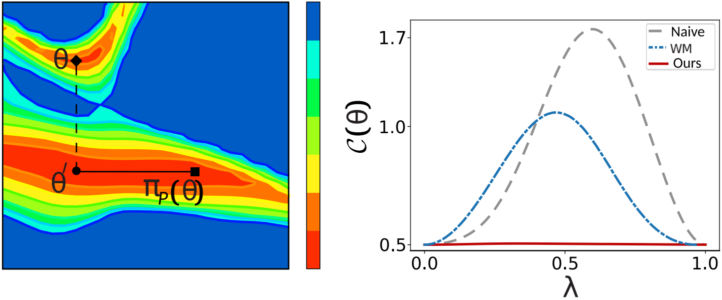



**Objective:** Find the permutation of $\theta$ that minimizes $\mathcal{C}_{Rnd}(\mathcal{P};\theta,\theta')$

$$\mathcal{C}_{Rnd}(\mathcal{P};\theta,\theta')=\mathcal{C}\left((1-\lambda)\theta'+\lambda\pi_{\mathcal{P}}(\theta)\right),\lambda\sim \mathrm{U}(0,1)$$

# Introduction

**Example:** Re-basin for linear mode connectivity




WM - Weight Matching re-basin [Ainsworth et al., 2022]

# Proposed method

# Permutation transformation

Let a feed forward neural network be defined as:

$f_\theta(x) = (\ell_h \circ \ldots \circ \ell_1)(x),$

$\ell_i(z) = \sigma(W_i z + b_i)$

a permuted model is defined as:

$f'_\theta(x) = (\ell'_h \circ \ldots \circ \ell'_1)(x),$

$\ell'_i(z) = \sigma(P_i W_i P_{i-1}^T z + P_i b_i),$

with $P_h = P_0^T = \boldsymbol{I}$

such that the functionally equivalence holds, $f_\theta(x) = f'_\theta(x), \forall x$

# Permutation transformation

$$f'_\theta(x) = (\ell'_h \circ \ldots \circ \ell'_1)(x),$$
$$\ell'_i(z) = \sigma(P_i W_i P_{i-1}^T z + P_i b_i),$$

where $P_i$ is a permutation matrix, i.e., a binary matrix such that there is only a single 1 per row and column.

$$P_i^T P_i = I$$

Example of permutation matrices:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

# Proposal – Re-basin via implicit Sinkhorn differentiation

Permuted model definition with soft permutation matrices

$$\ell_i'(z) = \sigma(S_\tau(P_i) W_i S_\tau(P_{i-1}^T) z + S_\tau(P_i) b_i)$$

with Sinkhorn operator [Mena et al., 2018] defined as:

$$S_\tau^{(0)}(X) = \exp\left(\frac{X}{\tau}\right),$$

$$S_\tau^{(t+1)}(X) = \mathcal{T}_c(\mathcal{T}_r(S_\tau^{(t)}(X))).$$

**Drawback:** Not efficient differentiation.

**Solution:** Implicit differentiation [Eisenberger et al., 2022]

# Proposal – Re-basin via implicit Sinkhorn differentiation

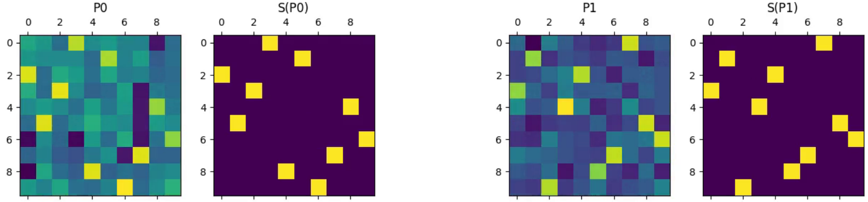



Figure: Soft permutation matrices before – $P_i$ – and after – $S(P_i)$ – applying Sinkhorn.

# Proposal – Re-basin via implicit Sinkhorn differentiation

Learning the permutations using Sinkhorn operator

**Model:** $\ell_i'(z) = \sigma(S_\tau(P_i) W_i S_\tau(P_{i-1}^T) z + S_\tau(P_i) b_i)$

**Cost functions:**

L2 distance loss: $\mathcal{C}_{L2}(\mathcal{P}; \theta_A, \theta_B) = ||\theta_A - \pi_{\mathcal{P}}(\theta_B)||^2$

Middle point loss: $\mathcal{C}_{Mid}(\mathcal{P}; \theta_A, \theta_B) = \mathcal{C}\left(\frac{\theta_A + \pi_{\mathcal{P}}(\theta_B)}{2}\right)$

Random point loss: $\mathcal{C}_{Rnd}(\mathcal{P}; \theta_A, \theta_B) = \mathcal{C}\left((1-\lambda)\theta_A + \lambda \pi_{\mathcal{P}}(\theta_B)\right)$, $\lambda \sim U(0,1)$

**Optimization problem:** $\mathcal{P}^* = \arg\min_{\mathcal{P}} \mathcal{C}(\mathcal{P}; \theta_A, \theta_B)$

Use with your favorite gradient descent-based algorithm!

# Proposal – Re-basin incremental learning

Cost function:

$$\mathcal{C}_{CL}(\delta_i, \mathcal{P}_i; \theta_i) = \mathcal{C}\left(\frac{\theta_i + \pi_{\mathcal{P}_i}(\theta_i)}{2} + \delta_i\right) + \beta||\delta_i||^2$$

Training process solves the optimization problem:

$$\delta_i^*, \mathcal{P}_i^* = \arg\min_{\delta_i, \mathcal{P}_i} \mathcal{C}_{CL}(\delta_i, \mathcal{P}_i; \theta_i)$$

After convergence we choose a model:

$$\theta_{i+1} = (1-\alpha)\theta_i + \alpha\pi_{\mathcal{P}_i}(\theta_i) + \delta_i$$

# Proposal – Re-basin via implicit Sinkhorn differentiation

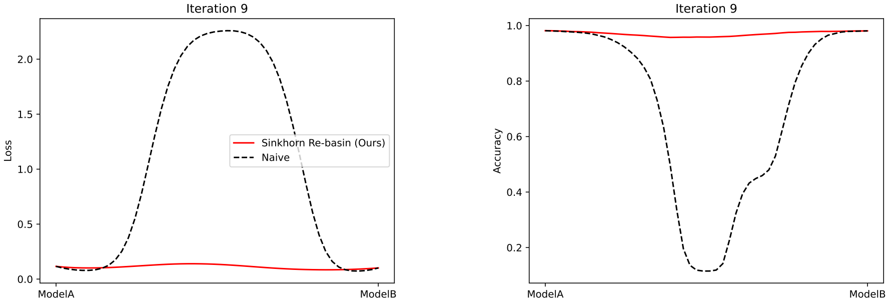
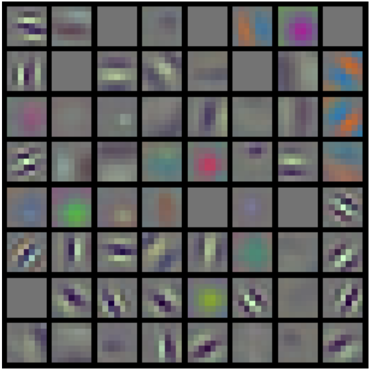



Figure: Linear mode connectivity using two VGG models trained over Mnist.

# Experimental results

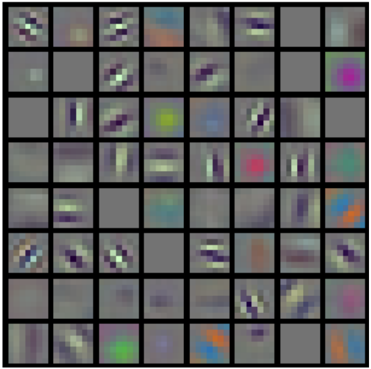
# Experiment 1 – Models alignment

**conv1** of pre-trained Resnet18



$\theta_A$

**conv1** of randomly permuted pre-trained Resnet18:



$\theta_B = \pi_{\mathcal{P}}(\theta_A)$

Can we find the permutations that align the models?

# Experiment 1 – Models alignment

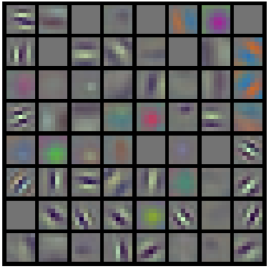
| Method | Init | 2 hidden ↓ | 4 hidden ↓ | 8 hidden ↓ |
|---|---|---|---|---|
| WM | Rnd | 6.05±9.17 | 4.12±6.58 | 0.50±1.55 |
| $\mathcal{C}_{L2}$ (Ours) | | 0.00±0.00 | 0.00±0.00 | 0.00±0.00 |
| WM | Pol3 | 0.57±2.84 | 0.07±0.46 | 0.01±0.10 |
| $\mathcal{C}_{L2}$ (Ours) | | 0.00±0.00 | 0.00±0.00 | 0.00±0.00 |
| WM | Pol1 | 0.27±0.94 | 0.00±0.00 | 0.00±0.00 |
| $\mathcal{C}_{L2}$ (Ours) | | 0.00±0.00 | 0.00±0.00 | 0.00±0.00 |

Table: L1 distance between the estimated and expected re-basing with different network initialization and depth. Distances are scaled $\times 10^3$.

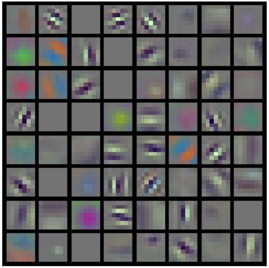
# Experiment 1 – Models alignment

Resnet18 models alignment

$\theta_A$



$\pi_{\mathcal{P}}(\theta_A)$



# Experiment 1 – Models alignment

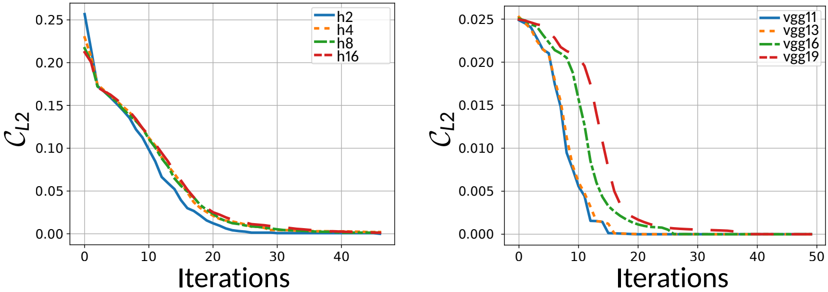
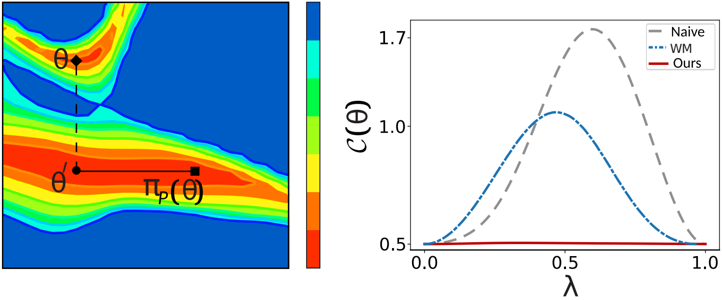



Figure: Validation loss during Sinkhorn re-basin training for feedforward neural networks with a different number of hidden layers (left panel) and VGG with increasing depth (right panel).

# Experiment 2 – Linear mode connectivity




Can we find the permutations that minimize the barrier in the linear path?

# Experiment 2 – Linear mode connectivity

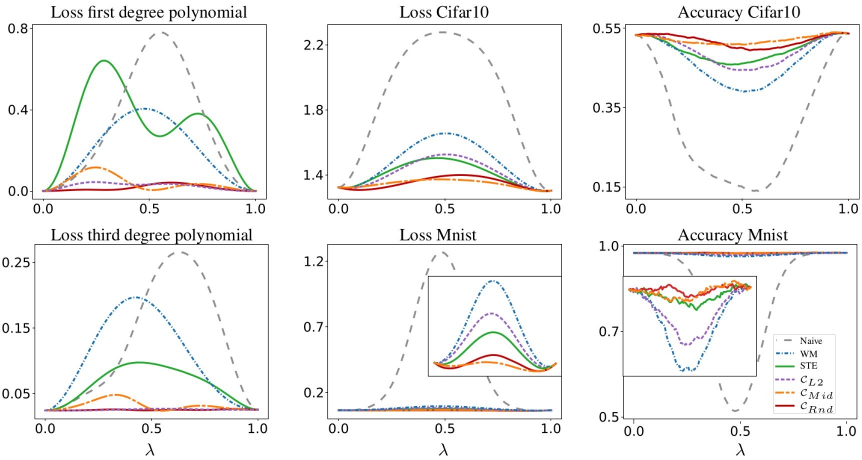



Figure: Linear mode connectivity achieved by WM [Ainsworth et al., 2022], STE [Ainsworth et al., 2022], and our Sinkhorn re-basin with $\mathcal{C}_{L2}$, $\mathcal{C}_{Mid}$, and $\mathcal{C}_{Rnd}$ costs for a NN with two hidden layers.

# Experiment 2 – Linear mode connectivity

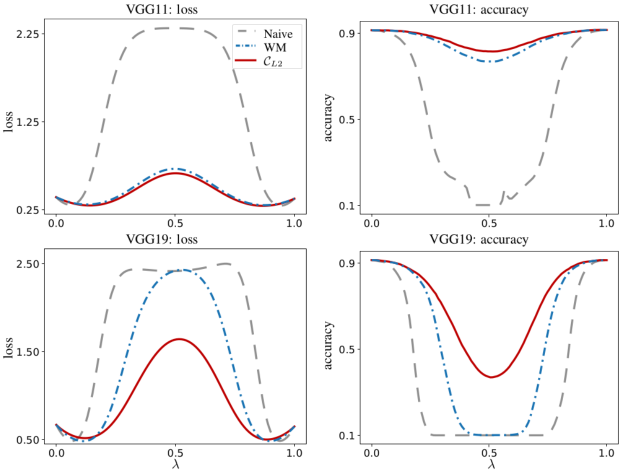



Figure: Linear mode connectivity using VGG11 and VGG19 networks trained over Cifar10 dataset.

# Experiment 2 – Linear mode connectivity

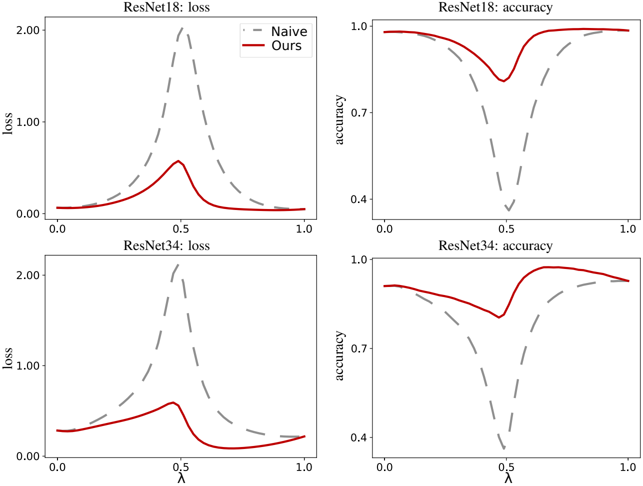
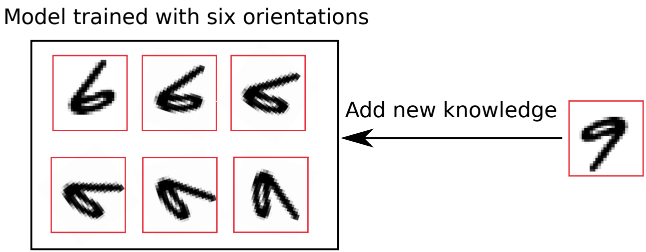



Figure: Linear mode connectivity using ResNet18 and ResNet34 trained over Imagenette dataset.

# Experiment 3 – Incremental learning




Can we find the permutation that allows us to finetune the model and add new knowledge while avoiding catastrophic forgetting?

# Experiment 3 – Incremental learning

| | Rotated Mnist | | Split Cifar100 | |
|---|---|---|---|---|
| Method | Accuracy ↑ | Forgetting ↓ | Accuracy ↑ | Forgetting ↓ |
| Finetune | 46.28±1.01 | 0.52±0.01 | 35.41±0.95 | 0.49±0.01 |
| EWC | 59.92±1.71 | 0.34±0.02 | 50.50±1.33 | 0.24±0.02 |
| LwF | 61.86±3.66 | 0.29±0.06 | 41.43±4.06 | 0.51±0.01 |
| A-GEM | 68.47±0.90 | 0.28±0.01 | 44.42±1.46 | 0.36±0.01 |
| Rebasin (Ours) | 78.14±0.50 | 0.12±0.01 | 51.34±0.74 | 0.07±0.02 |
| Joint training | 90.84±4.30 | 0.00 | 60.48±0.54 | 0.00 |

Table: Performance of our proposed and state-of-art methods on incremental learning benchmarks over 20 episodes.

# Experiment 3 – Incremental learning

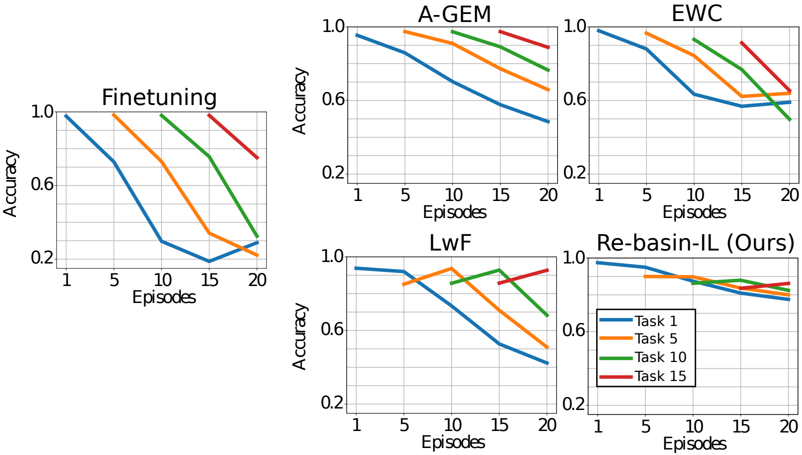



Figure: Evolution of task's accuracy during the incremental learning experience on Rotated Mnist with 20 episodes. Only tasks 1, 5, 10, and 15 are shown.

# Conclusions

# Conclusions

- Gradient descent-based re-basin
- Easy to adapt to new objectives
- New loss functions for neuron alignment, linear mode connectivity, and incremental learning
- Simple to use (PyTorch only!)

```
from rebasin import RebasinNet
modelA = MLP(input_size = 28 * 28, num_classes = 10)
pi_modelA = RebasinNet(modelA, input_shape = (1, 28 * 28))
```

# Thank you!

Visit our project website!

[Ainsworth et al., 2022] Ainsworth, S. K., Hayase, J., and Srinivasa, S. (2022).
Git re-basin: Merging models modulo permutation symmetries.
*arXiv preprint arXiv:2209.04836*.

[Eisenberger et al., 2022] Eisenberger, M., Toker, A., Leal-Taixé, L., Bernard, F., and Cremers, D. (2022).
A unified framework for implicit sinkhorn differentiation.
In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 509–518.

[Mena et al., 2018] Mena, G., Belanger, D., Linderman, S., and Snoek, J. (2018).
Learning latent permutations with gumbel-sinkhorn networks.
*arXiv preprint arXiv:1802.08665*.