E2PN: Efficient SE(3)-Equivariant Point Network

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Overview

SE(3) equivariant	Point-cloud network	KPConv style

What is it?

Light-weight	Fast	Easy to use

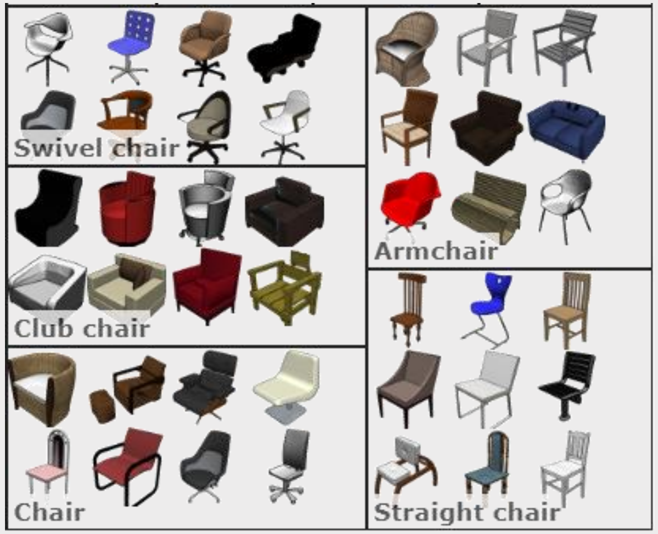
What are the advantages?

Quotient-space convolution	Permutation layer	Symmetric kernel

How is it done?

Background

Shape variations



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Pose variations

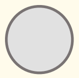
- Data augmentation
- Equivariant models

* Chang, Angel X., et al. "Shapenet: An information-rich 3d model repository." *arXiv preprint arXiv:1512.03012* (2015).

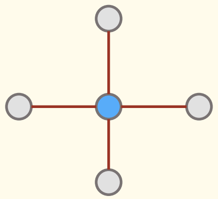
Equivariant model review: Group CNN

- Conventional convolution
  - $f: \mathbb{R}^n \rightarrow \mathbb{R},\ \kappa: \mathbb{R}^n \rightarrow \mathbb{R}$
  - $[\kappa * f](x) = \int_{\mathbb{R}^n} \kappa(t) f(x+t) \mathrm{d}t$
  - $[\kappa * f]: \mathbb{R}^n \rightarrow \mathbb{R}$

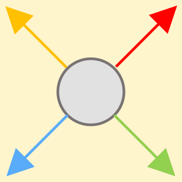
$x \in \mathbb{R}^3$



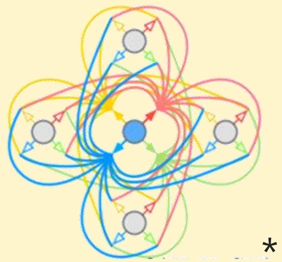
Conv at x



- Group convolution[1]: lifting the domain
  - $f: G \rightarrow \mathbb{R},\ \kappa: G \rightarrow \mathbb{R}$
  - $[\kappa * f](x) = \int_{G} \kappa(g) f(g \cdot x) \mathrm{d}g$
  - $[\kappa * f]: G \rightarrow \mathbb{R}$



$x \in \mathrm{SO}(3) \times \mathbb{R}^3$



Conv at x

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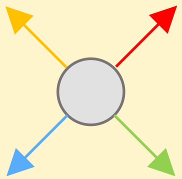
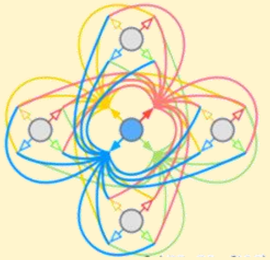
* Chen, Haiwei, et al. "Equivariant point network for 3d point cloud analysis." *CVPR*. 2021.

Improving the efficiency

- Group convolution[1]: lifting the domain
  - $f\colon G \to \mathbb{R},\ \kappa\colon G \to \mathbb{R}$
  - $[\kappa * f](x) = \int_G \kappa(g) f(g \cdot x)\mathrm{d}g$
  - $[\kappa * f]\colon G \to \mathbb{R}$

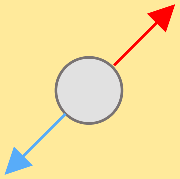


$x \in \mathrm{SO}(3) \times \mathbb{R}^3$

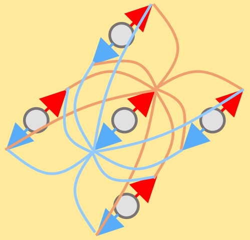


Conv at x

- Our method: quotient space convolution
  - lifting the domain to a quotient space
  - $f\colon G/H \to \mathbb{R},\ \kappa\colon G/H \to \mathbb{R}$
  - $[\kappa * f](x) = \int_{G/H} \kappa(g) f(g \cdot x)\mathrm{d}g$
  - $[\kappa * f]\colon G/H \to \mathbb{R}$
  - $|G/H| < |G|$



$x \in G/H$



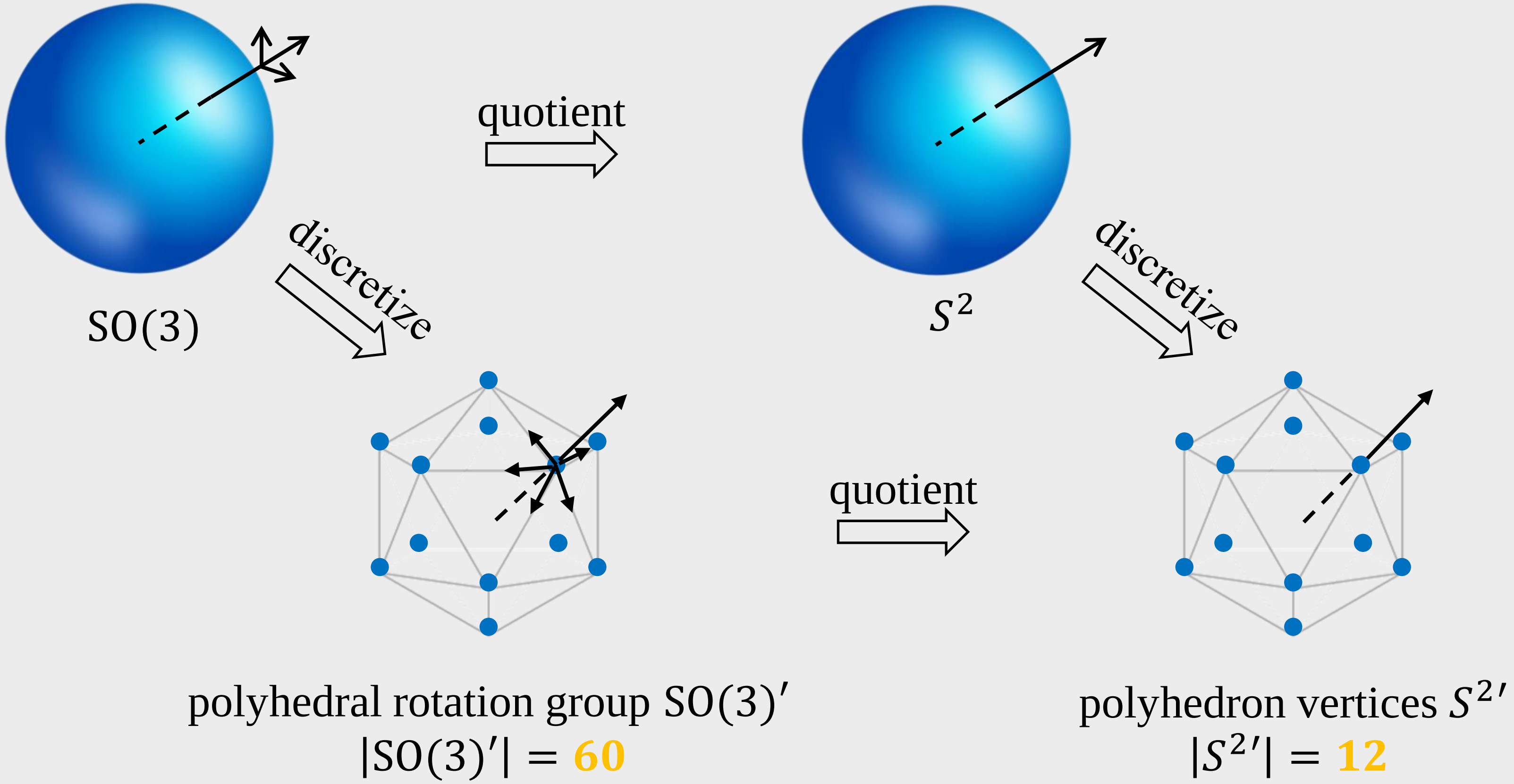
Conv at x

- In our specific setup:
  - $G = \mathrm{SE}(3) = \mathrm{SO}(3) \times \mathbb{R}^3, H = \mathrm{SO}(2)$
  - $G/H = S^2 \times \mathbb{R}^3$

Quotient space feature map

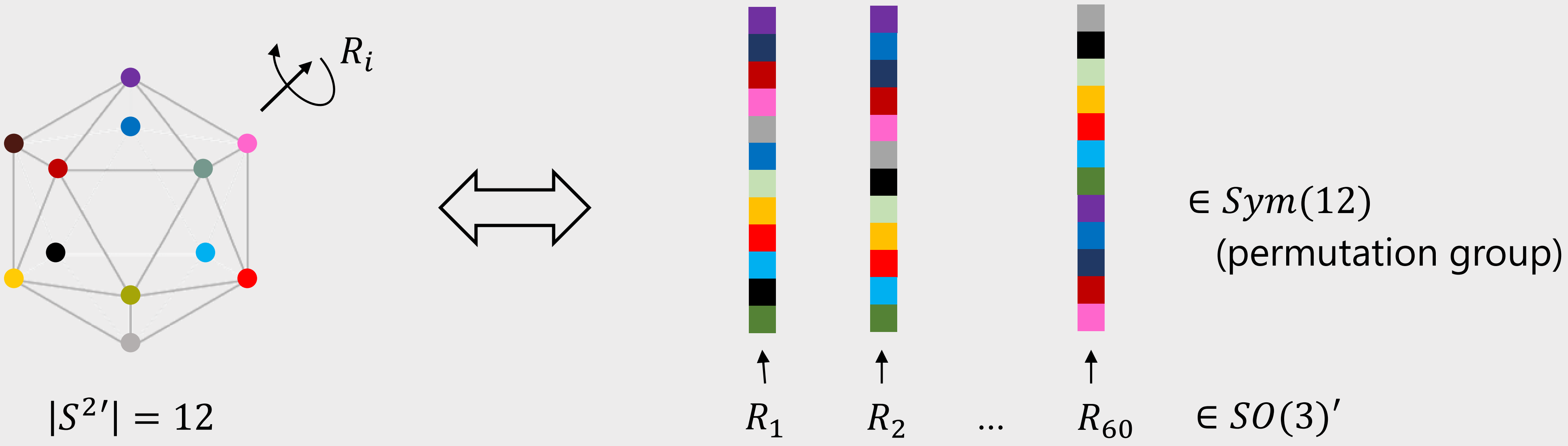
Our feature map is defined on $S^2 \times \mathbb{R}^3$ (**5-DOF**) instead of $SO(3) \times \mathbb{R}^3$ (**6-DOF**).

$$S^2 = SO(3)/SO(2)$$

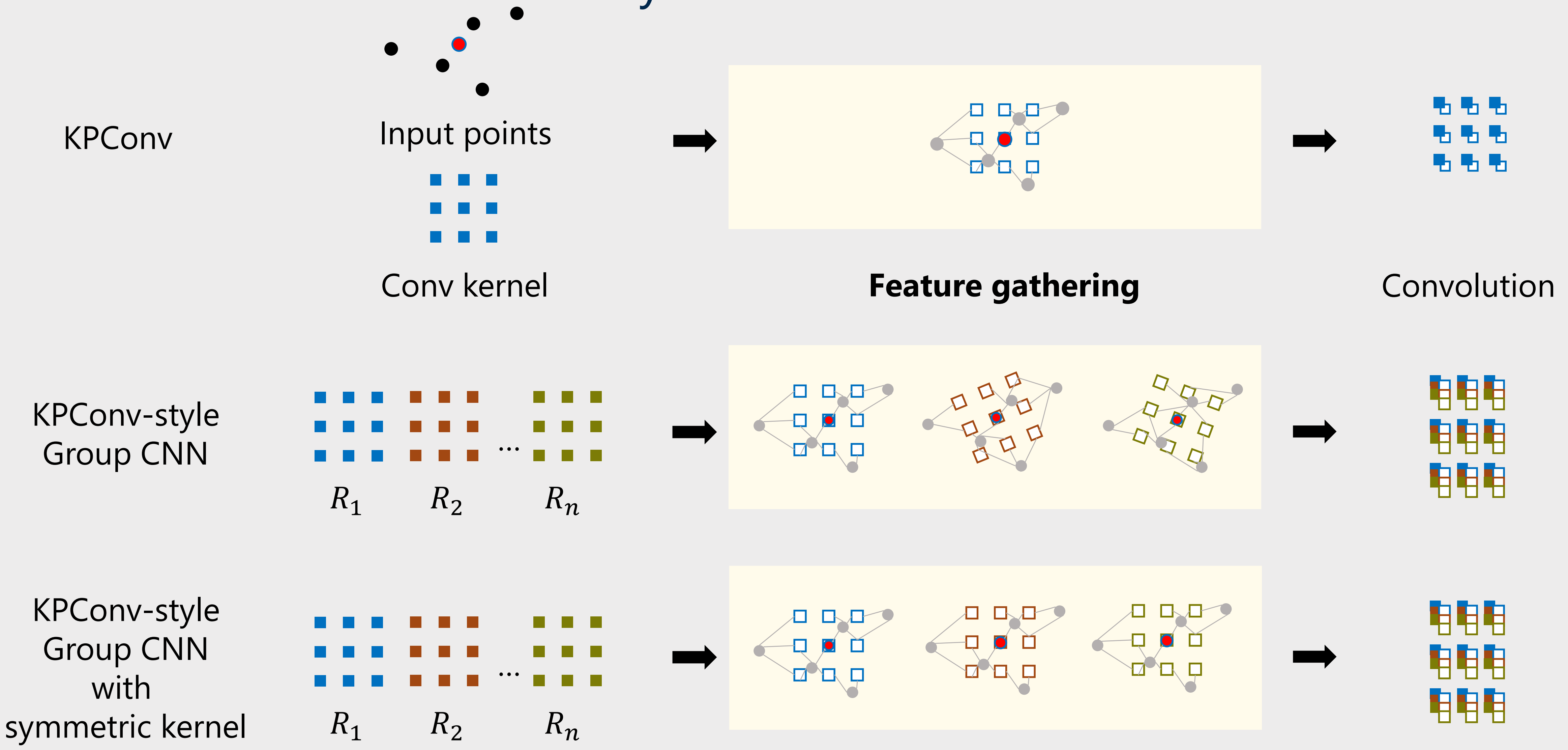


Recover $SO(3)$ information from S^2 features

Each element in $SO(3)'$ corresponds to a unique permutation of $S^{2\prime}$ (polyhedral vertices).

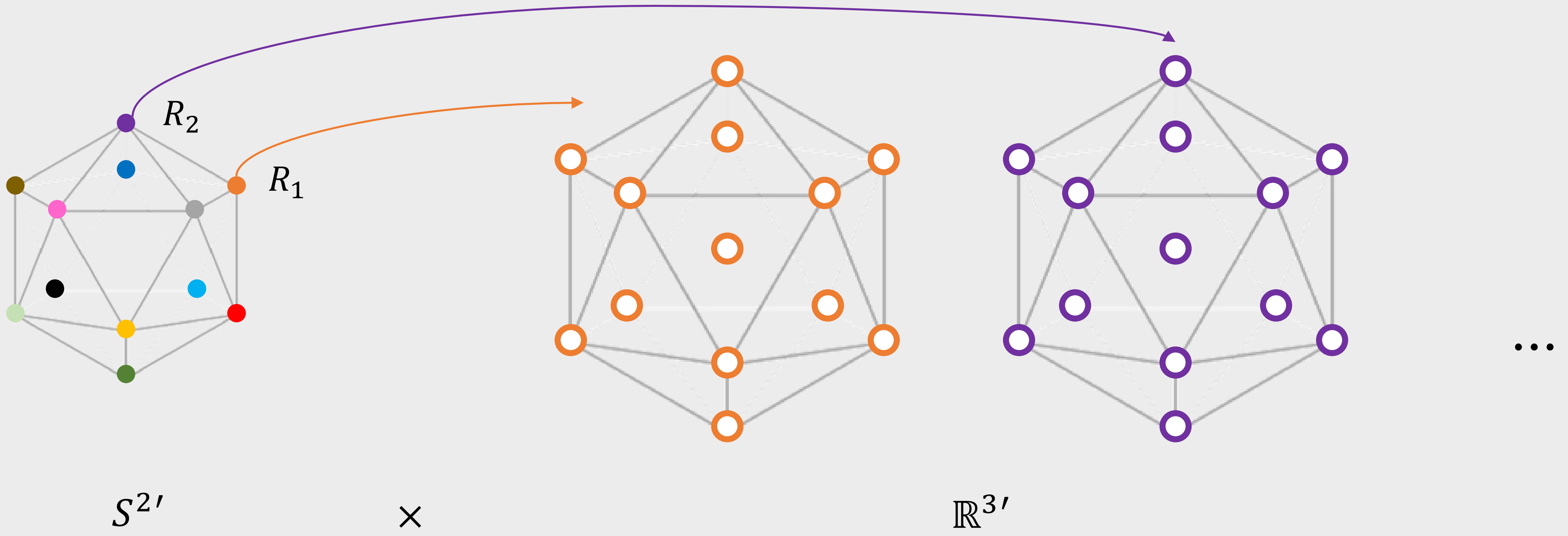


Symmetric kernel



Symmetric kernel

We use the polyhedron vertices as the kernel points.



The kernel after rotations lands in the same set of points.

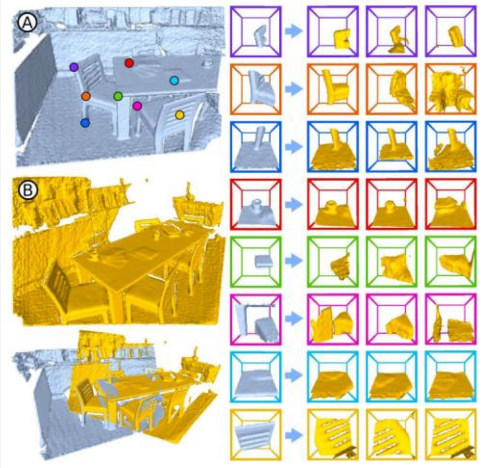
Experiments

ModelNet40

Tasks:

Object classification

Pose estimation



3DMatch

Task:

Keypoint Matching

Memory consumption

Speed

Tasks	ModelNet40 Pose		ModelNet40 Classification		3DMatch Keypoint Matching	
Methods	Memory (GB) ↓	Speed (fps) ↑	Memory (GB) ↓	Speed (fps) ↑	Memory (GB) ↓	Speed (fps) ↑
EPN (Group CNN baseline)	22.2 / 16.9	1.1 / 1.6	13.4 / 12.7	1.9 / 1.5	37.4 / 8.5	0.6 / 3.1
E2PN (*ours*)	**4.3 / 2.8**	**6.7 / 11.1**	**3.9 / 2.7**	**9.1 / 10.3**	**6.5 / 2.4**	**3.7 / 23.6**

Experiments

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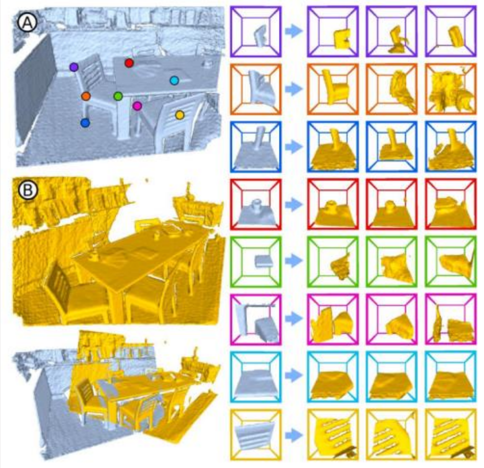

3DMatch

Task:

Keypoint Matching

Performance on par with Group CNN.

	Acc (%) ↑
EPN	84.63
E2PN (*ours*)	**86.99**

Object classification

	Avg μ (°) ↓	SD σ (°) ↓
EPN	**1.10**	0.20
E2PN (*ours*)	1.20	**0.08**

Pose estimation

	Recall (%) ↑
EPN	**97.6**
E2PN (*ours*)	97.3

Keypoint Matching

Conclusion

- A new SE(3)-equivariant network for 3D point clouds.
- Drastically improved efficiency compared with existing equivariant models.
- Simple structure, directly applicable to any network with a KPConv-style backbone.

https://github.com/minghanz/E2PN