# TUE-PM-070

# Revisiting the P3P Problem

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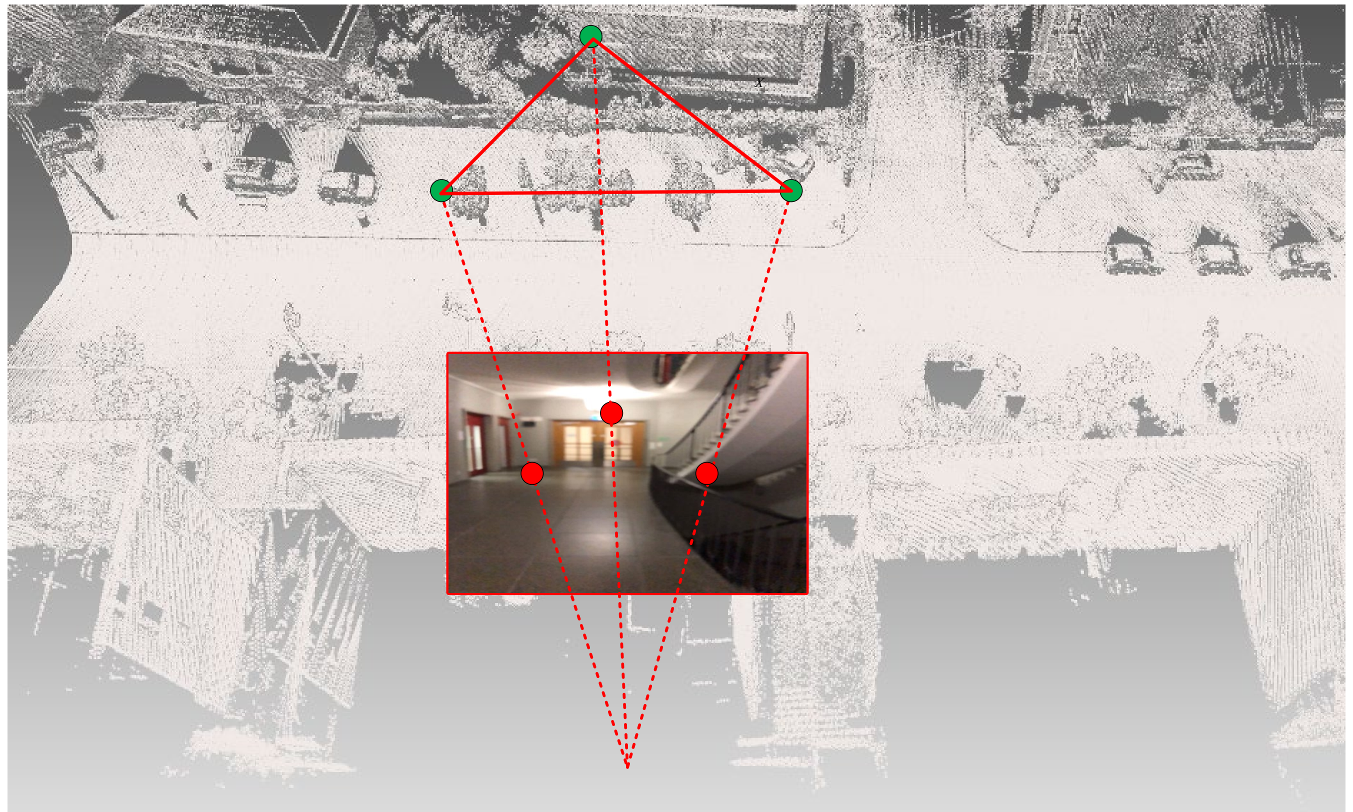
# The P3P (Perspective-Three-Point) Problem

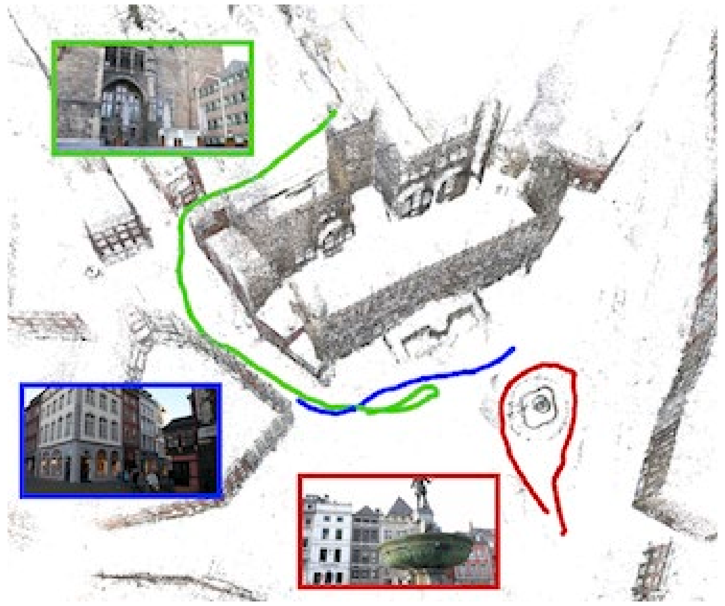


Estimating the absolute pose of a calibrated camera from three 3D-to-2D correspondences is a fundamental problem in computer vision.

**Applications**

- Odometry
- 3D reconstruction
- SLAM
- AR & VR
- Autonomous Driving
- …



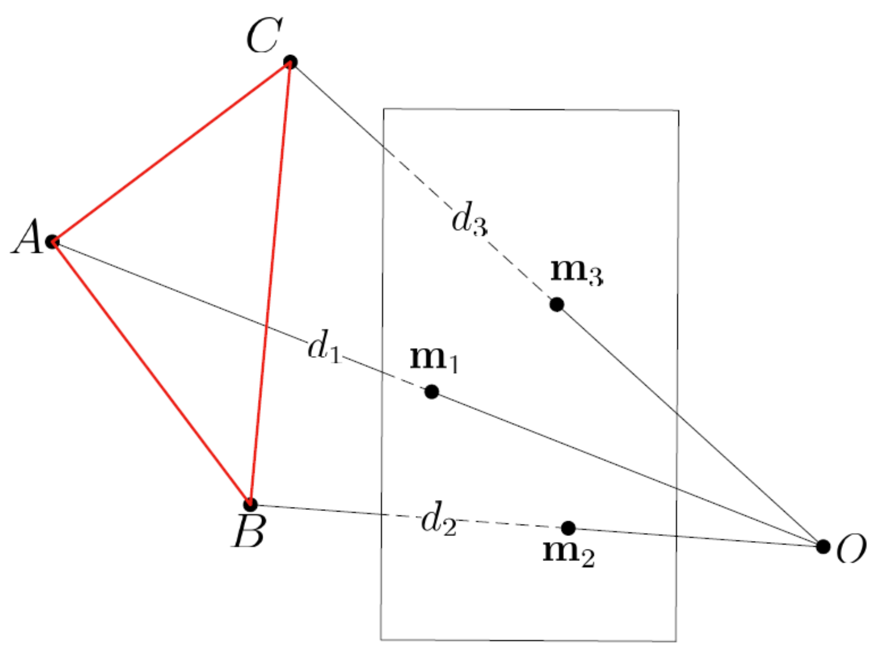
# Motivation and Preliminaries

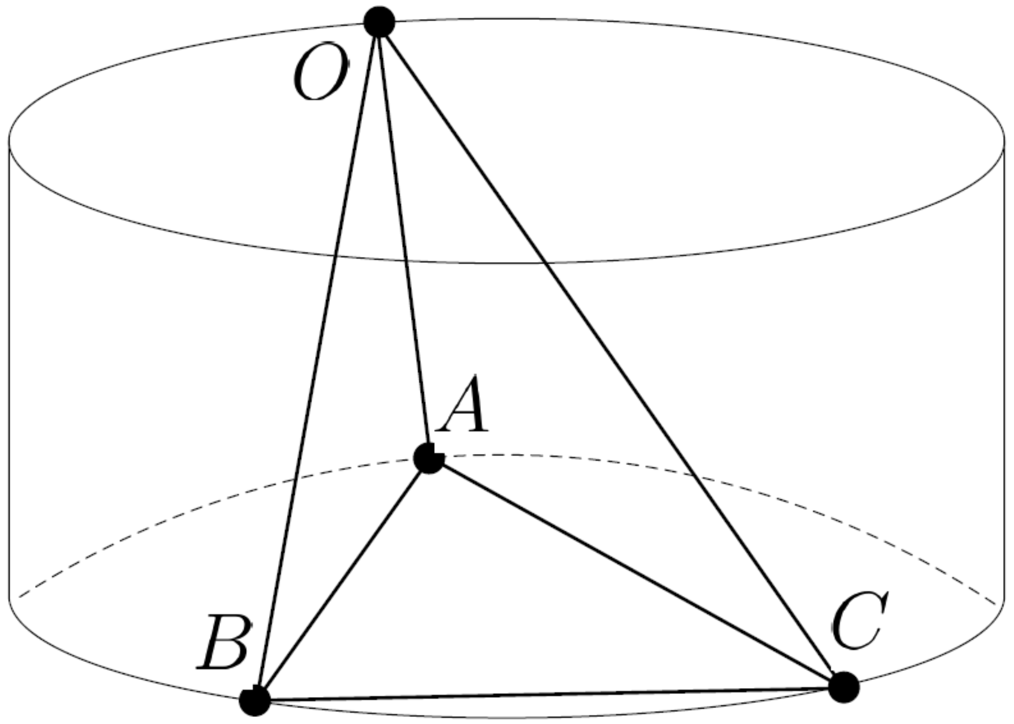


While the current state-of-the-art solvers are both extremely fast and stable, there still exist configurations where they may break down.

➢ Danger Cylinder

- It has been shown in [1] that the solution of the P3P problem is unstable if the optical center O lies on the surface of this danger cylinder.




The danger cylinder is defined as a circular cylinder circumscribing points A,B,C with axis normal to the plane ABC

[1]. EH Thompson. Space resection: Failure cases. The Photogrammetric Record, 5(27):201–207, 1966

# Contributions



- We focus on the solution strategy that is based on intersecting two conics
- We provide a fast and stable solver based on the characterization of the possible solution configurations
- leveraging our new understanding we design a novel P3P algorithm that explicitly handles the dangerous cases.

# Problem Formulation

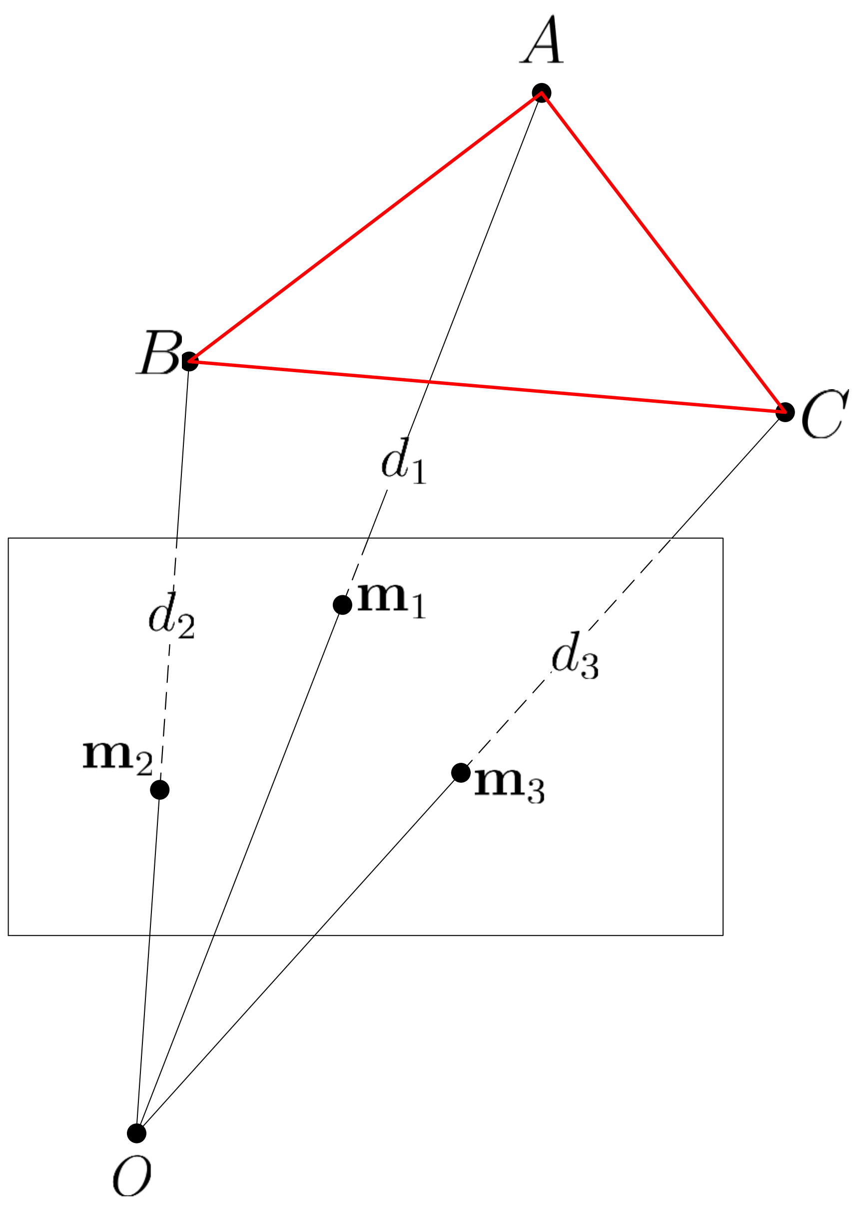


The 3D-2D points are related by the following transformation

$$d_i \mathbf{m}_i = \mathbf{R}\mathbf{X}_i + \mathbf{t}$$

Assuming $|\mathbf{m}_i| = 1, i \in \{1, 2, 3\}$, and using the law of cosines we obtain the following constraints

$$d_1^2 + d_2^2 - 2d_1 d_2 \mathbf{m}_1^\top \mathbf{m}_2 = |AB|^2,$$
$$d_1^2 + d_3^2 - 2d_1 d_3 \mathbf{m}_1^\top \mathbf{m}_3 = |AC|^2,$$
$$d_2^2 + d_3^2 - 2d_2 d_3 \mathbf{m}_2^\top \mathbf{m}_3 = |BC|^2,$$




Then we have the following two quadratic equations in two unknowns x and y by eliminating $d_3$

$$x^2 + (1-a)y^2 - 2m_{12}xy + 2am_{23}y - a = 0,$$
$$x^2 - by^2 - 2m_{13}x + 2bm_{23}y + 1 - b = 0,$$

where $x = d_1/d_3,\ y = d_2/d_3$ . Now the P3P problem is reduced to find the real solutions of the above two quadratic equations.

# Our Approach



The two quadratic equation can be written as the following matrix representations

$$[1, x, y]\mathbf{C}_1[1, x, y]^\top = 0,$$
$$[1, x, y]\mathbf{C}_2[1, x, y]^\top = 0,$$

We first construct a 3x3 matrix

$$\mathbf{C} = \mathbf{C}_1 + \sigma\mathbf{C}_2.$$

If the matrix C is not of full rank, then the conic is termed degenerate. Degenerate point conics are either two lines (rank 2) or a repeated line (rank 1), and can be written as

$$\mathbf{C} = \mathbf{p}\mathbf{q}^\top + \mathbf{q}\mathbf{p}^\top,$$

- Finding the Degenerate Conic

Since the conic should be degenerate, we have

$$\begin{aligned} f(\sigma) &= \det(\mathbf{C}) \\ &= \det(\mathbf{C}_1 + \sigma\mathbf{C}_2) = 0, \end{aligned}$$

# Our Approach



Extracting the lines from the degenerate conic

➢ Direct method

Assuming we have obtained a degenerate conic **C**, which can be written as

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}.$$

Since $\mathbf{C} = \mathbf{p}\mathbf{q}^\top + \mathbf{q}\mathbf{p}^\top$ , let $\mathbf{p} = [p_1, p_2, p_3]^\top, \mathbf{q} = [q_1, q_2, q_3]^\top$,
the matrix C can also be written as

$$\mathbf{C} = \begin{bmatrix} 2p_1q_1 & p_1q_2 + p_2q_1 & p_1q_3 + p_3q_1 \\ p_1q_2 + p_2q_1 & 2p_2q_2 & p_2q_3 + p_3q_2 \\ p_1q_3 + p_3q_1 & p_2q_3 + p_3q_2 & 2p_3q_3 \end{bmatrix}.$$

# Our Approach



Extracting the lines from the degenerate conic

➢ Finding the intersection of two lines

We can also recover the intersection point $\mathbf{v} = \mathbf{p} \times \mathbf{q}$, which can then be used to extract the lines from **C**. Once we obtain the intersection point **v**, the skew-symmetric matrix of **v** is given by

$$[\mathbf{v}]_\times = \mathbf{p}\mathbf{q}^\top - \mathbf{q}\mathbf{p}^\top,$$

Then we define a new matrix

$$\mathbf{D} = \mathbf{C} + [\mathbf{v}]_\times.$$

We can find that $\mathbf{D} = 2\mathbf{p}\mathbf{q}^\top$. The pair of lines $\{\mathbf{p}, \mathbf{q}\}$ can be found from one row and the corresponding column of the matrix **D**.

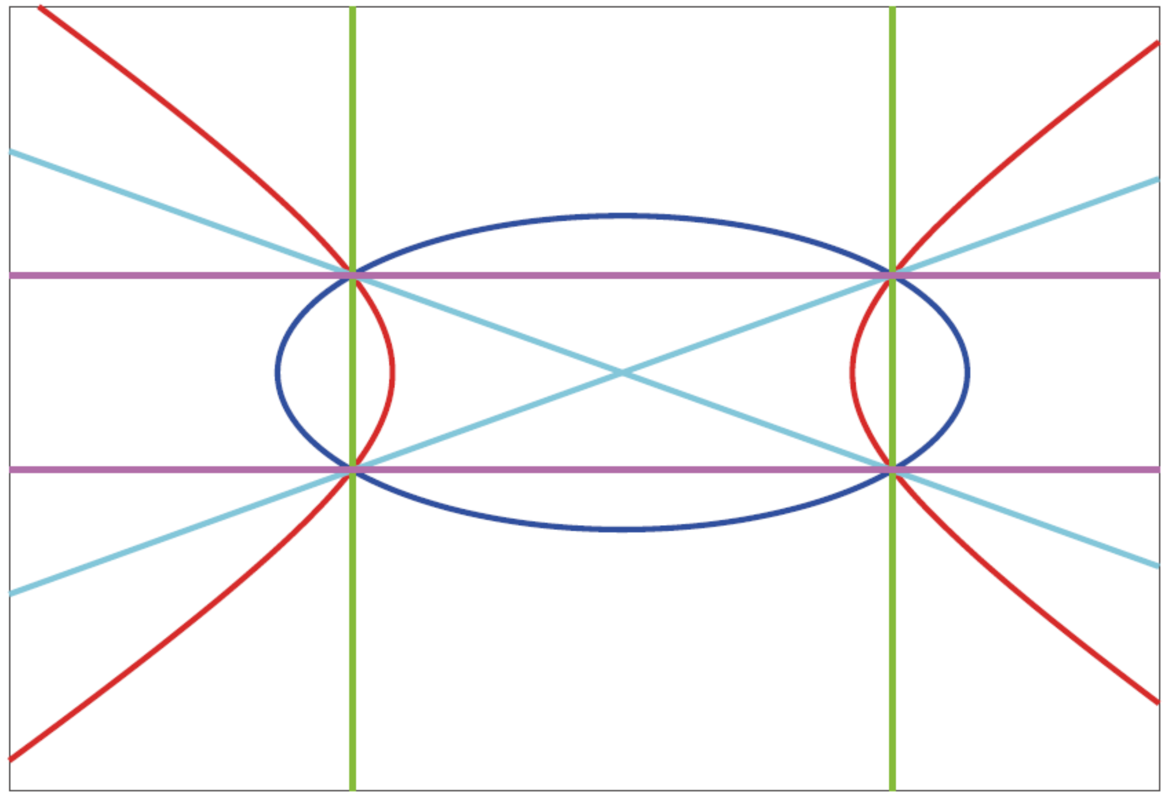
**Rank-1 case**: If the degenerate conic **C** includes a pair of repeated lines, the matrix **C** will be rank-1. In this case, the repeated lines can be recovered directly from one row or column.

# Possible Solution Configurations



| Case | Roots of the cubic | | Number of lines | | Intersections of each pair of lines: real | imaginary |
|---|---|---|---|---|---|---|
| a | simple real | 3 | simple real | 1 pair | 0 | 4 |
| | imaginary | 0 | imaginary | 2 pairs | 0 | 4 |
| b | simple real | 3 | simple real | 3 pairs | 4 | 0 |
| | imaginary | 0 | imaginary | 0 | - | - |
| c | simple real | 1 | simple real | 1 pair | 2 | 2 |
| | imaginary | 2 | imaginary | 2 pairs | 2 | 2 |
| d | simple real | 1 | simple real | 1 pair | 1D | 2 |
| | double real | 1 | imaginary | 1 pair (twice) | 1D | 2 |
| e | simple real | 1 | simple real | 1 pair | 2D | 0 |
| | double real | 1 | repeated real | 1 pair (twice) | 2D | 0 |
| f | simple real | 1 | simple real | 1 pair | 1D + 2 | 0 |
| | double real | 1 | simple real | 1 pair (twice) | 1D + 2 | 0 |
| g | Triple root | 1 | simple real | 1 pair (thrice) | 1T + 1 | 0 |
| h | Triple root | 1 | repeated real | 1 pair (thrice) | 1Q | 0 |

Four intersections



The relationship among the roots of the cubic equation, the number of the lines from the degenerate conic and the intersections of the two conics.
1D, 1T and 1Q denote one double, one triple and one quadruple intersection. Case (d)-(h) are critical cases with $\Delta = 0$, where $\Delta$ is the discriminant of the cubic equation.

# Possible Solution Configurations



By discussing the relative position of the two conics, we are able to analytically characterize the real roots of the polynomial system and employ a tailored solution strategy for each problem instance.

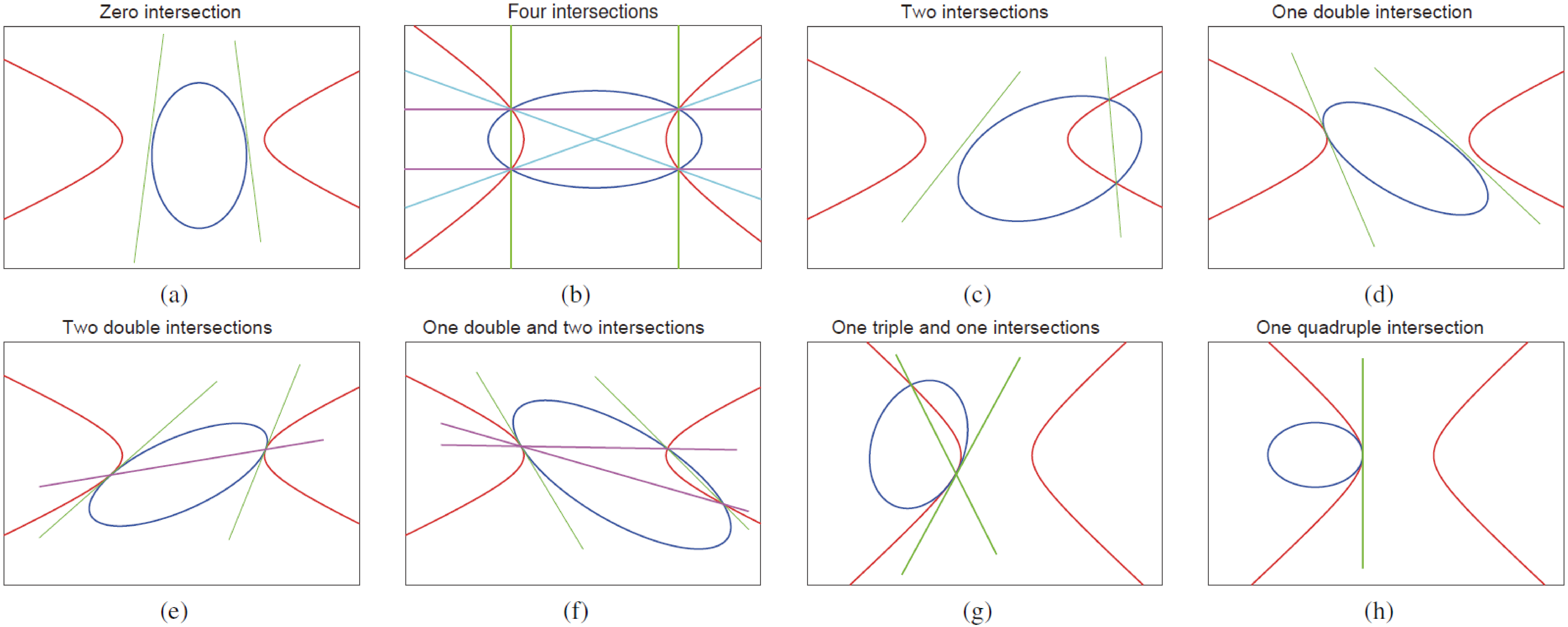



Illustration of eight possible cases for the relative position of a hyperbola and an ellipse. The pair of lines with different colors corresponds to different cubic roots.

# Experimental Results



| Method | Ours | Persson *et al.* [21] | Ke *et al.* [14] | Kneip *et al.* [15] | Nakano [19] | Nakano(rp) [19] |
|---|---|---|---|---|---|---|
| Valid solutions | 16825700 | 16825700 | 17389005 | 24159054 | 16823126 | 16826586 |
| Unique solutions | 16825700 | 16825686 | 16850758 | 16827917 | 16815718 | 16826042 |
| Duplicates | 0 | 0 | 163038 | 3038 | 0 | 0 |
| Good solution | 10000000 | 9999989 | 9999622 | 9999663 | 9996957 | 9999249 |
| No solution | 0 | 11 | 378 | 337 | 3043 | 751 |
| Ground truth | 9999993 | 9999978 | 9997345 | 9991078 | 9985342 | 9998727 |
| Incorrect | 0 | 2 | 375209 | 7328099 | 7408 | 544 |

Table 1. Solution comparison with the current state-of-the-art solvers

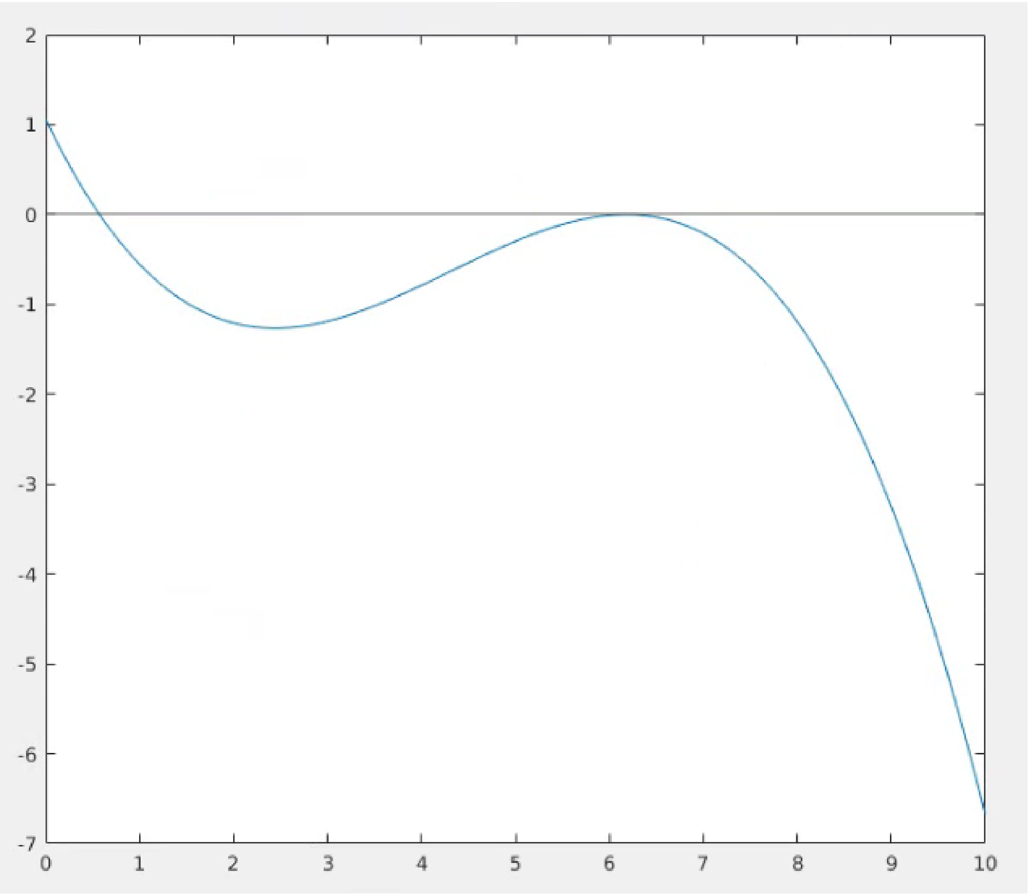
| Timing (ns) | Ours | Persson *et al.* [21] | Ke *et al.* [14] | Kneip *et al.* [15] | Nakano [19] | Nakano(rp) [19] |
|---|---|---|---|---|---|---|
| Mean | **225.8** | 260.6 | 387.1 | 667.2 | 591.3 | 702.0 |
| Median | **225.7** | 260.7 | 387.1 | 667.3 | 591.0 | 702.1 |
| Min | **224.0** | 258.2 | 384.4 | 664.1 | 588.1 | 699.4 |
| Max | **231.6** | 263.7 | 393.5 | 670.7 | 611.8 | 705.5 |
| Speed up | **1.154** | 1.0 | 0.6732 | 0.3906 | 0.4407 | 0.3712 |

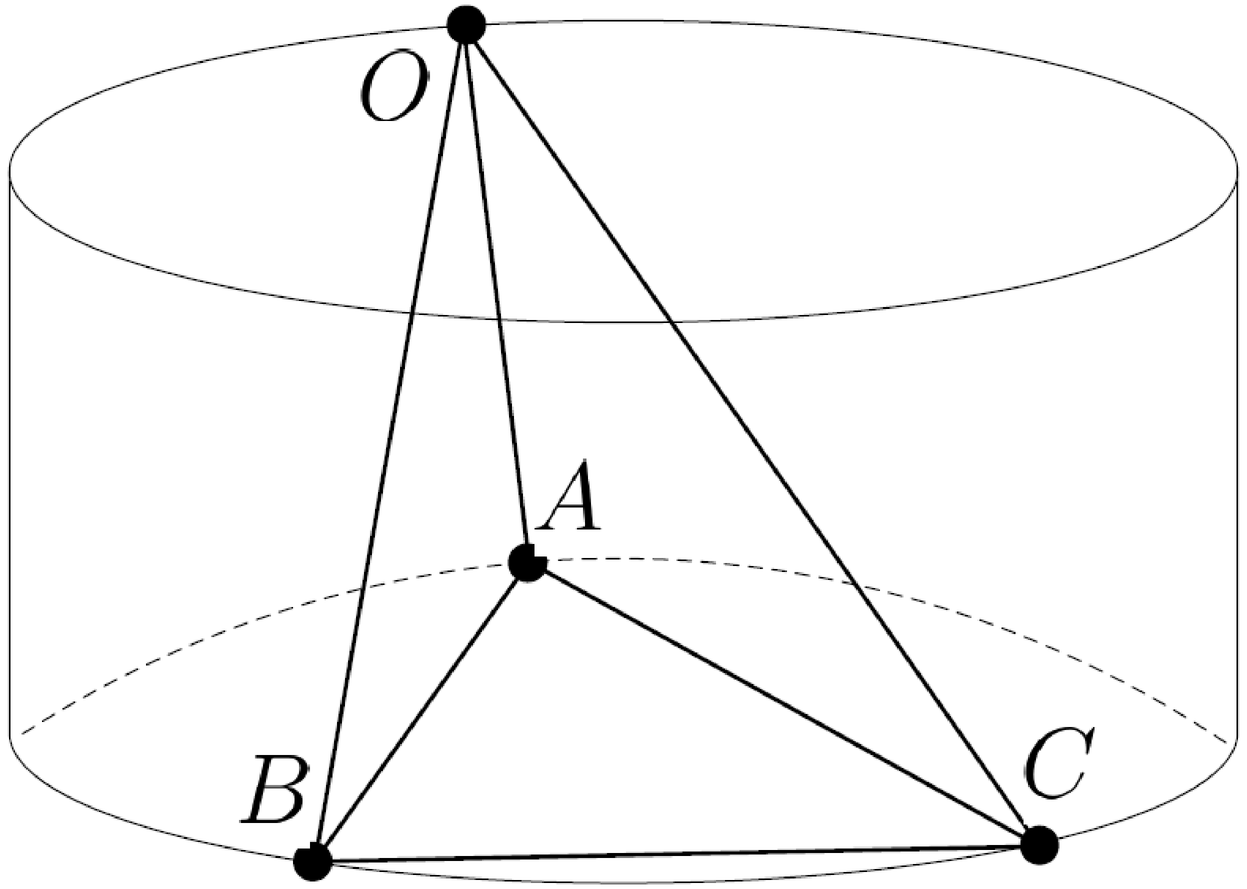
Table 2. Running times comparison

# Relationship to the Danger Cylinder



The three 3D points A,B,C defines a cylinder with the generatrix parallel to the normal of the plane ABC. This cylinder is known as the danger cylinder in the literature [1].

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**Thank you for your attention!**