Exact-NeRF: An Exploration of a Precise Volumetric Parameterization for Neural Radiance Fields

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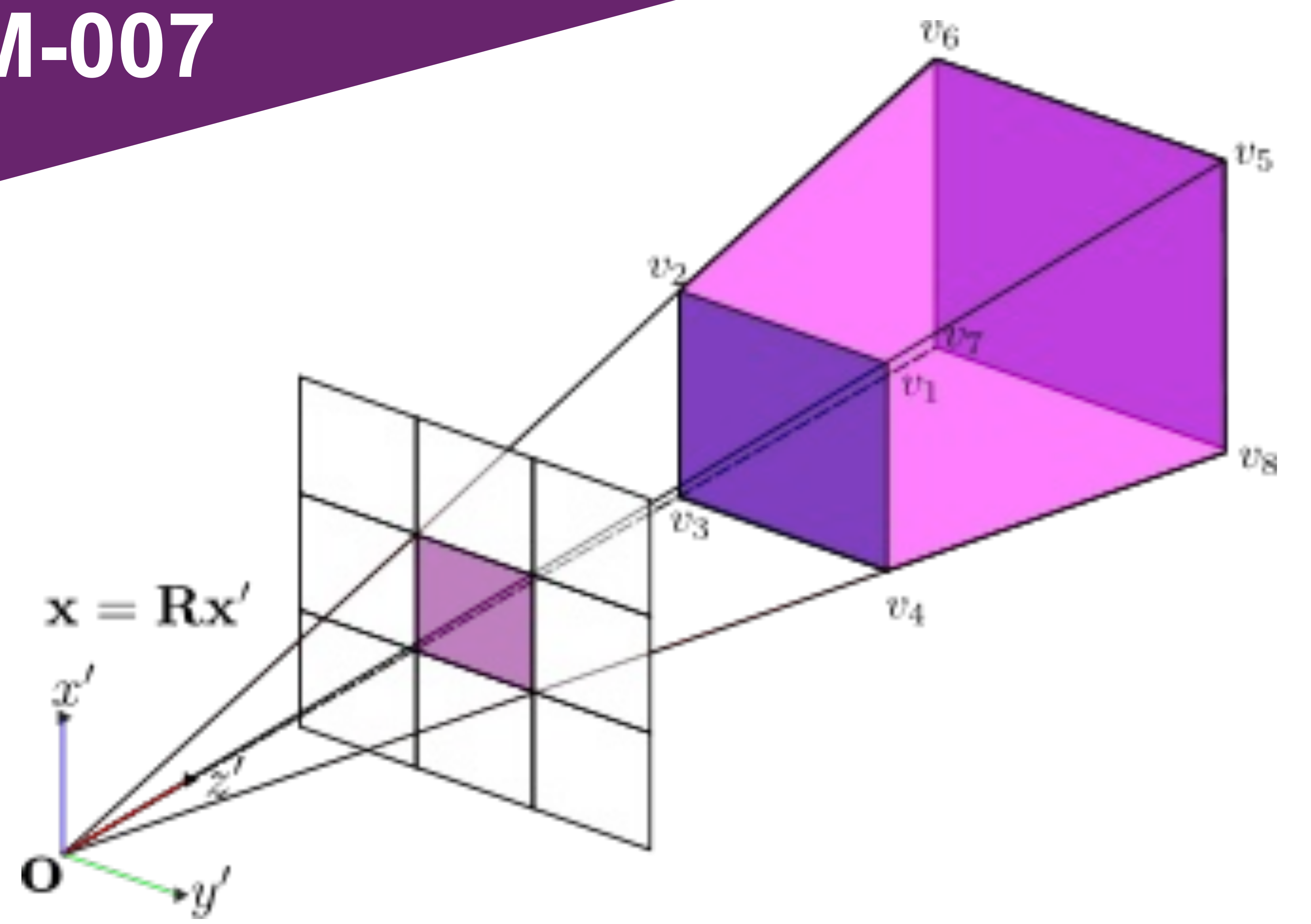

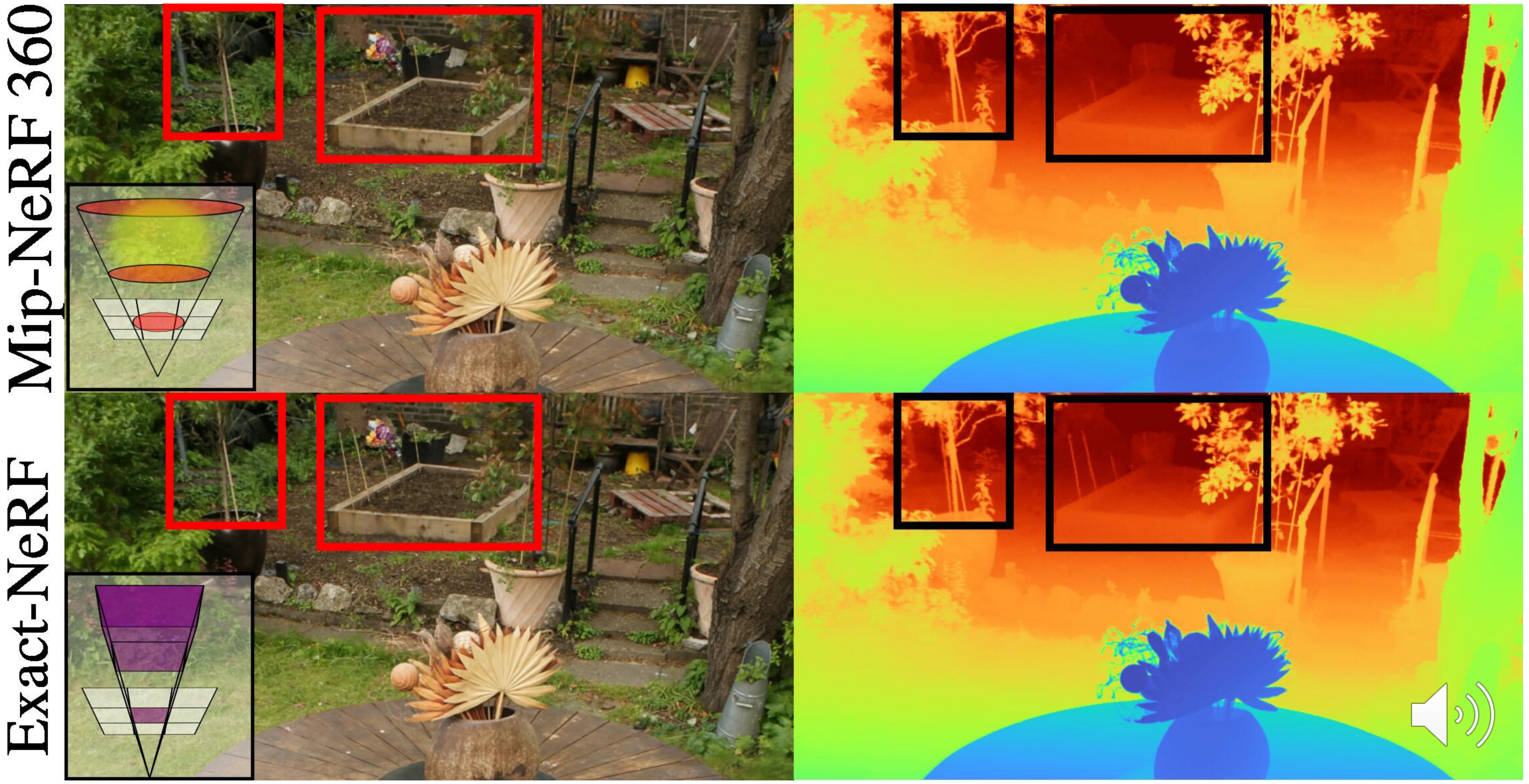

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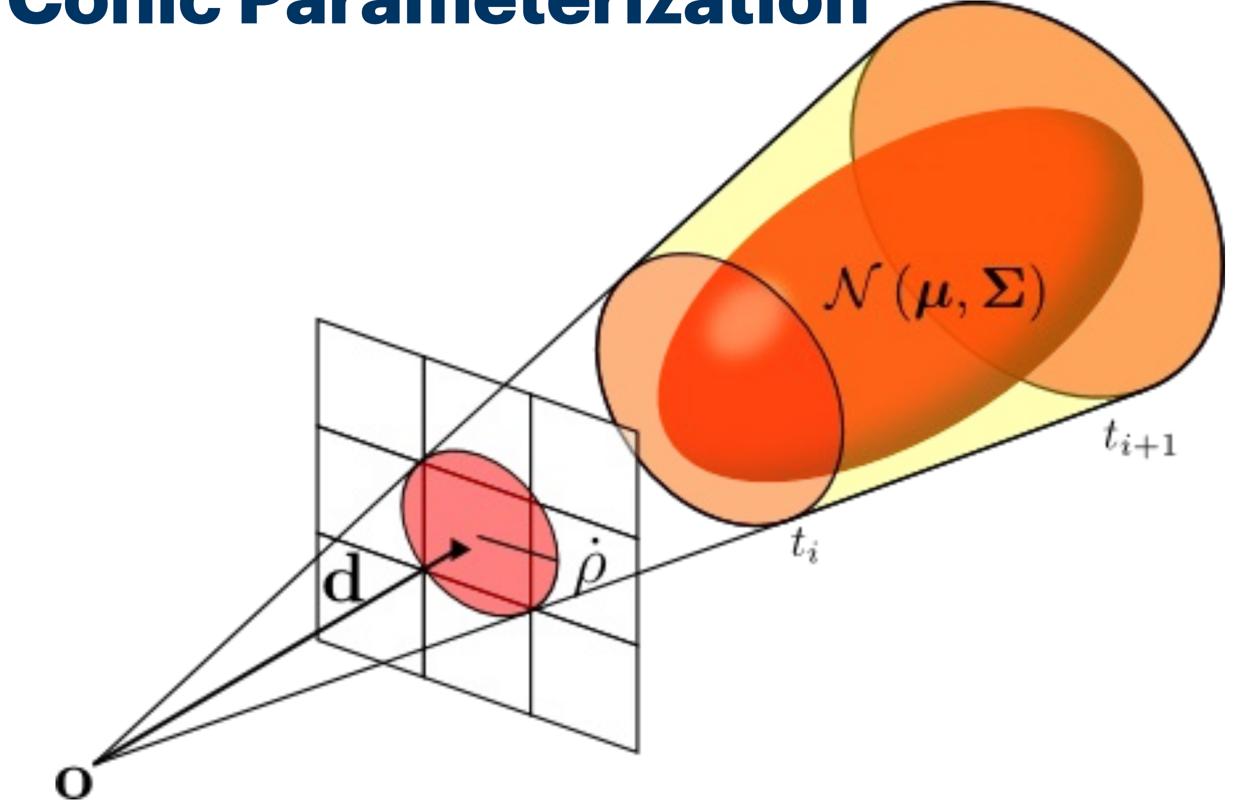
Conic Parameterization



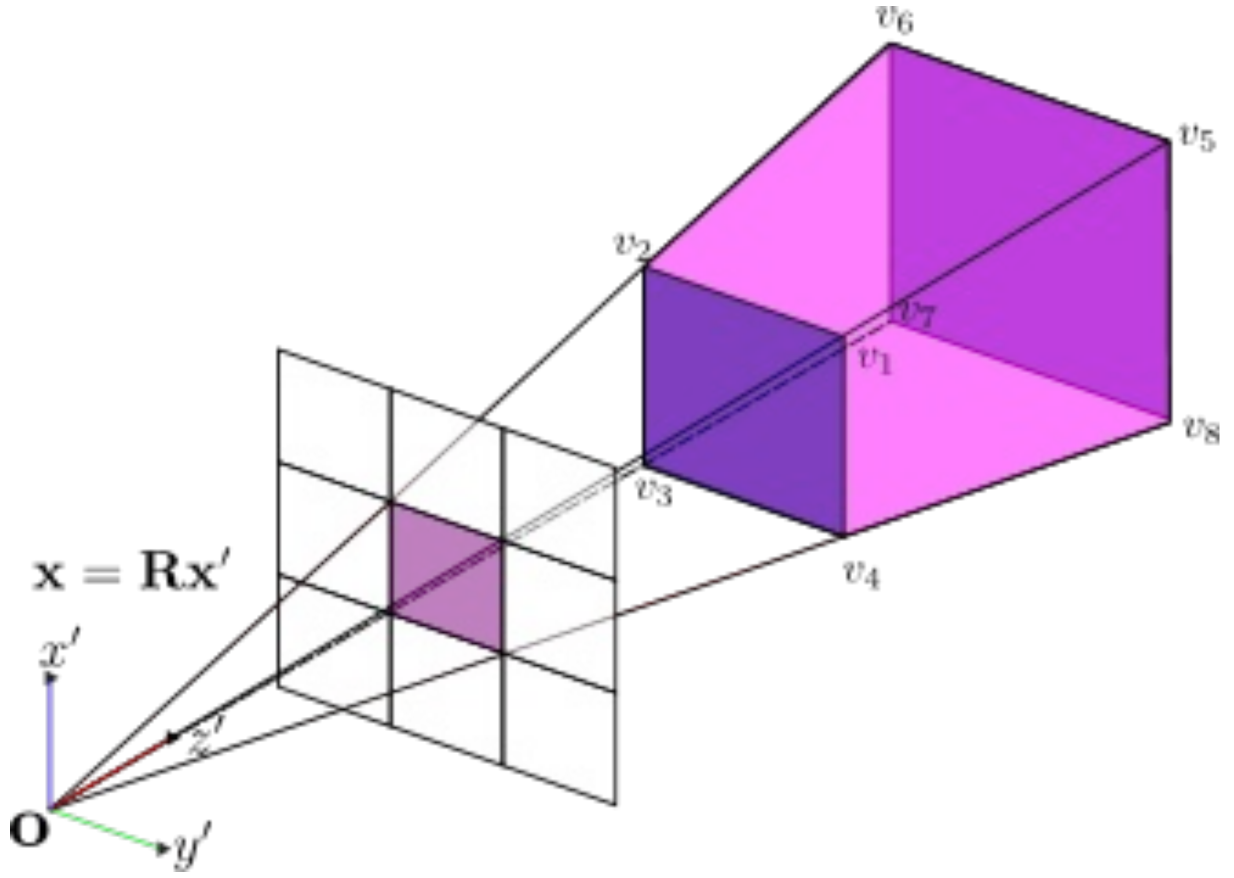
Integrated Positional Encoding (IPE)

$$\gamma_I(\mathbf{d},\mathbf{o},\dot{\rho},t_i,t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$

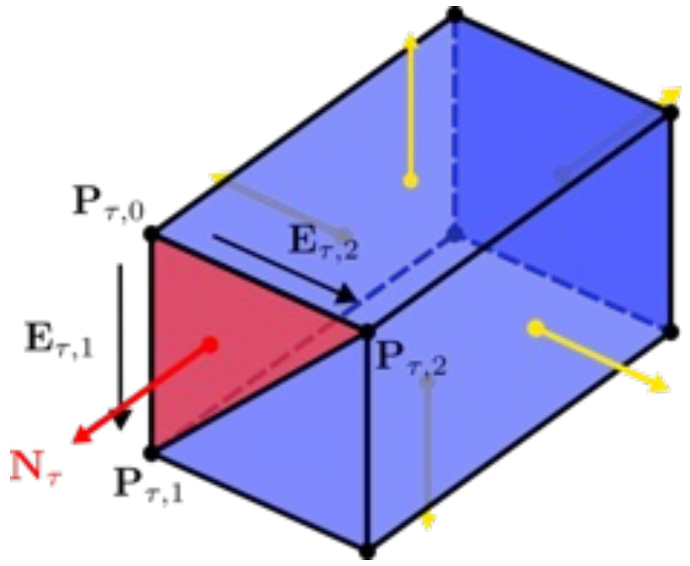
Mip-NeRF Approximation

$$\gamma_I^*(\boldsymbol{\mu},\boldsymbol{\Sigma}) = \mathbb{E}_{\mathbf{x}\sim\mathcal{N}(\mathbf{P}\boldsymbol{\mu},\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^\top)}[\gamma(\mathbf{x})]$$

Pyramid Parameterization



Exact-NeRF Positional Encoding



$$\iiint \nabla\cdot\mathbf{F}dV = \oint_{\partial S}\mathbf{F}\cdot d\mathbf{S}$$

$$\iiint_F \sin(2^l\mathbf{x}_k)dV = \frac{1}{2^{3l}}\sum_{\tau\in\mathcal{T}}\sigma_{k,\tau}\mathbf{N}_\tau\cdot\mathbf{e}_k$$

$$\iiint_F dV = \frac{1}{6}\sum_{\tau\in\mathcal{T}}\mathbf{P}_{\tau,0}\cdot\mathbf{N}_\tau$$

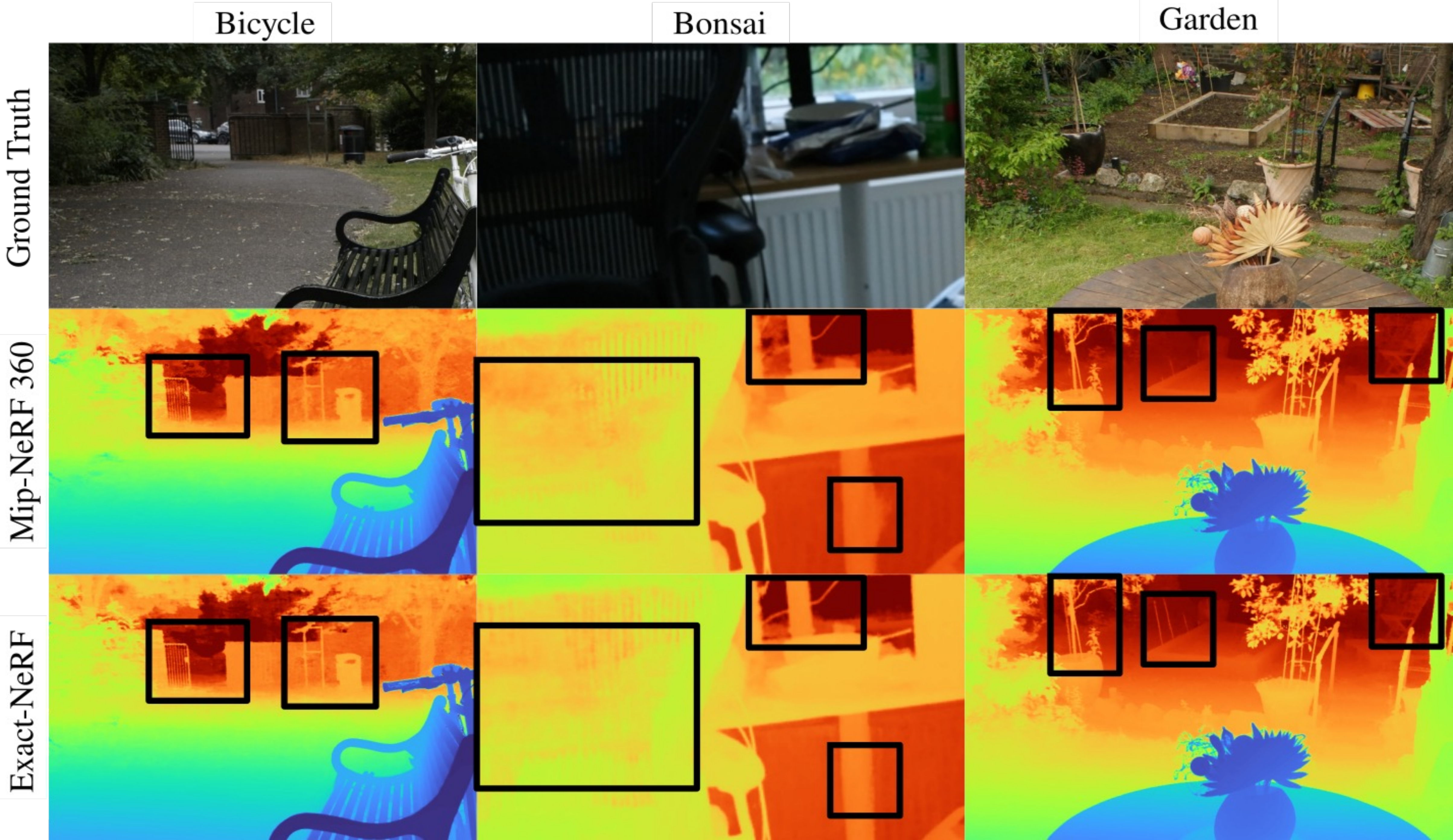
Exact-NeRF is competitive with mip-NeRF, with better background depth estimation.

Blender Dataset

Model	PSNR ↑	SSIM ↑	LPIPS ↓	DISTS ↓	Avg ↓
Mip-NeRF	**34.766**	**0.9706**	0.0675	0.0822	**0.0242**
Exact-NeRF	34.707	0.9705	**0.0667**	**0.0878**	**0.0242**

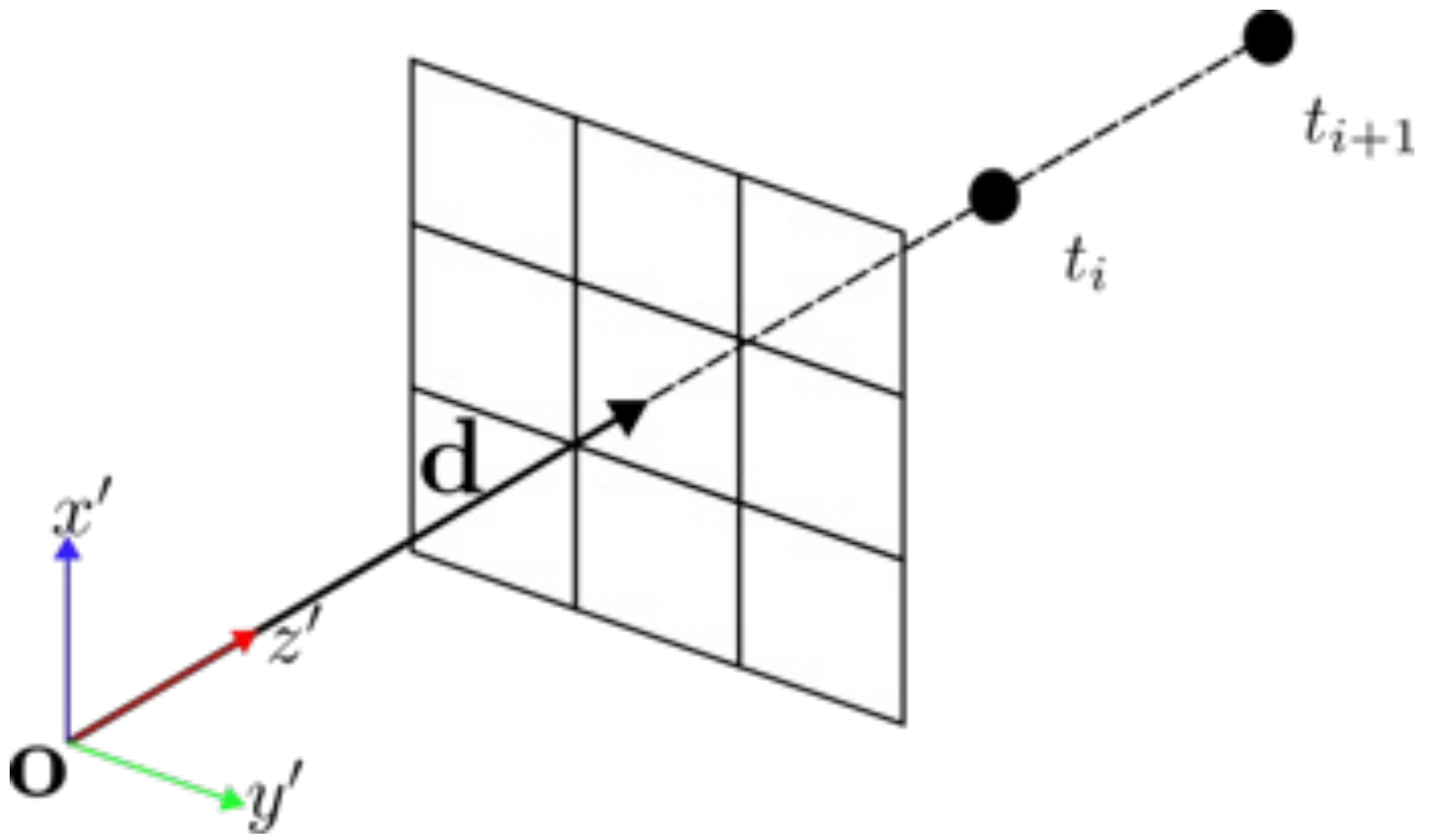
Mip-NeRF 360 Dataset

Model	PSNR ↑	SSIM ↑	LPIPS ↓	DISTS ↓	Avg ↓
Mip-NeRF 360	**27.325**	**0.7942**	**0.6559**	**0.2438**	**0.1077**
Exact-NeRF	27.230	0.7881	0.6569	0.2452	0.1088



Motivation

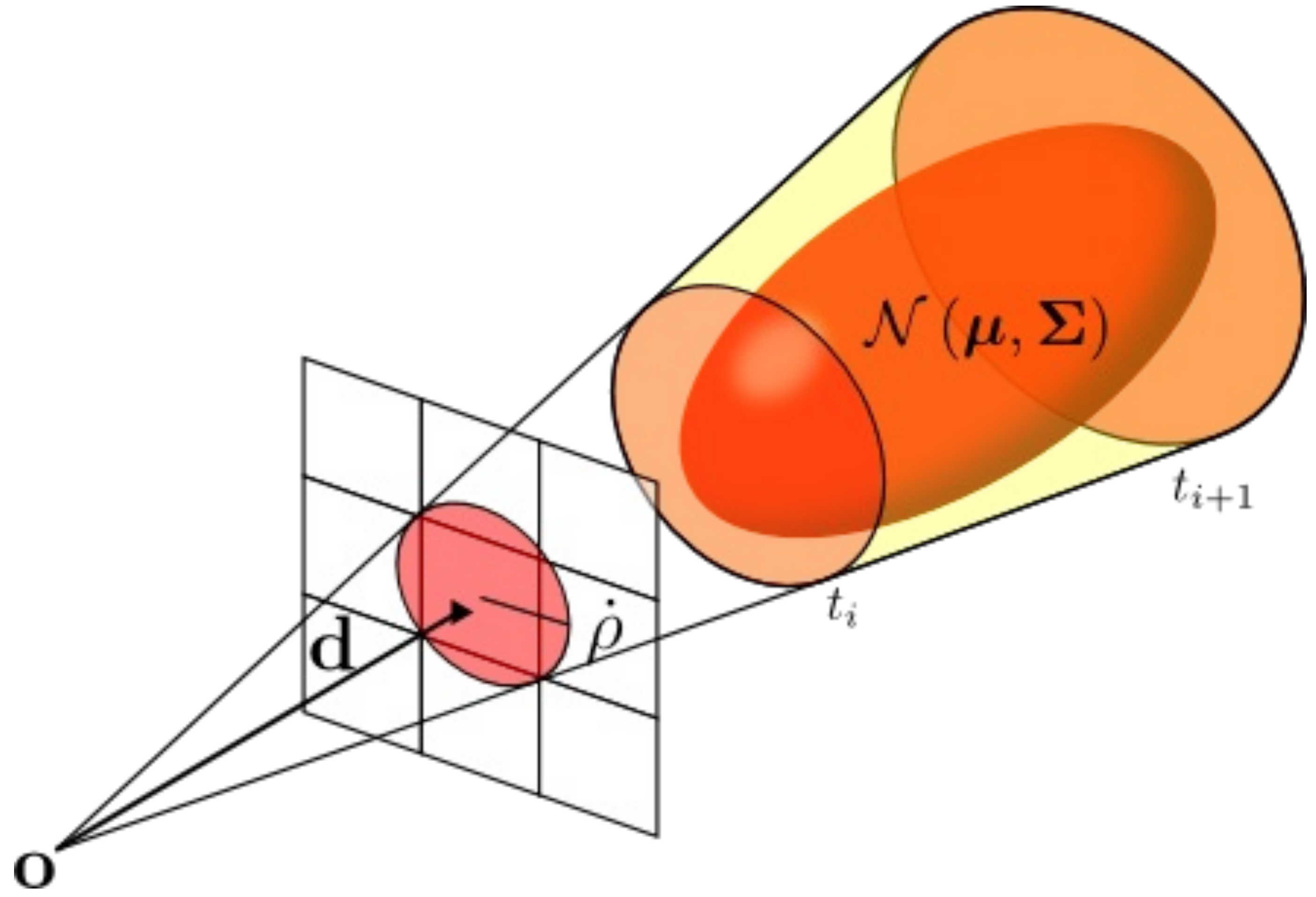
NeRF: Rays and point sampling



Positional Encoding

$$\gamma(\mathbf{x}) = [\sin(2^0\mathbf{x}), \cos(2^0\mathbf{x}), \ldots, \sin(2^{L-1}\mathbf{x}), \cos(2^{L-1}\mathbf{x})]$$

Mip-NeRF: Cones and volumetric sampling



Integrated Positional Encoding

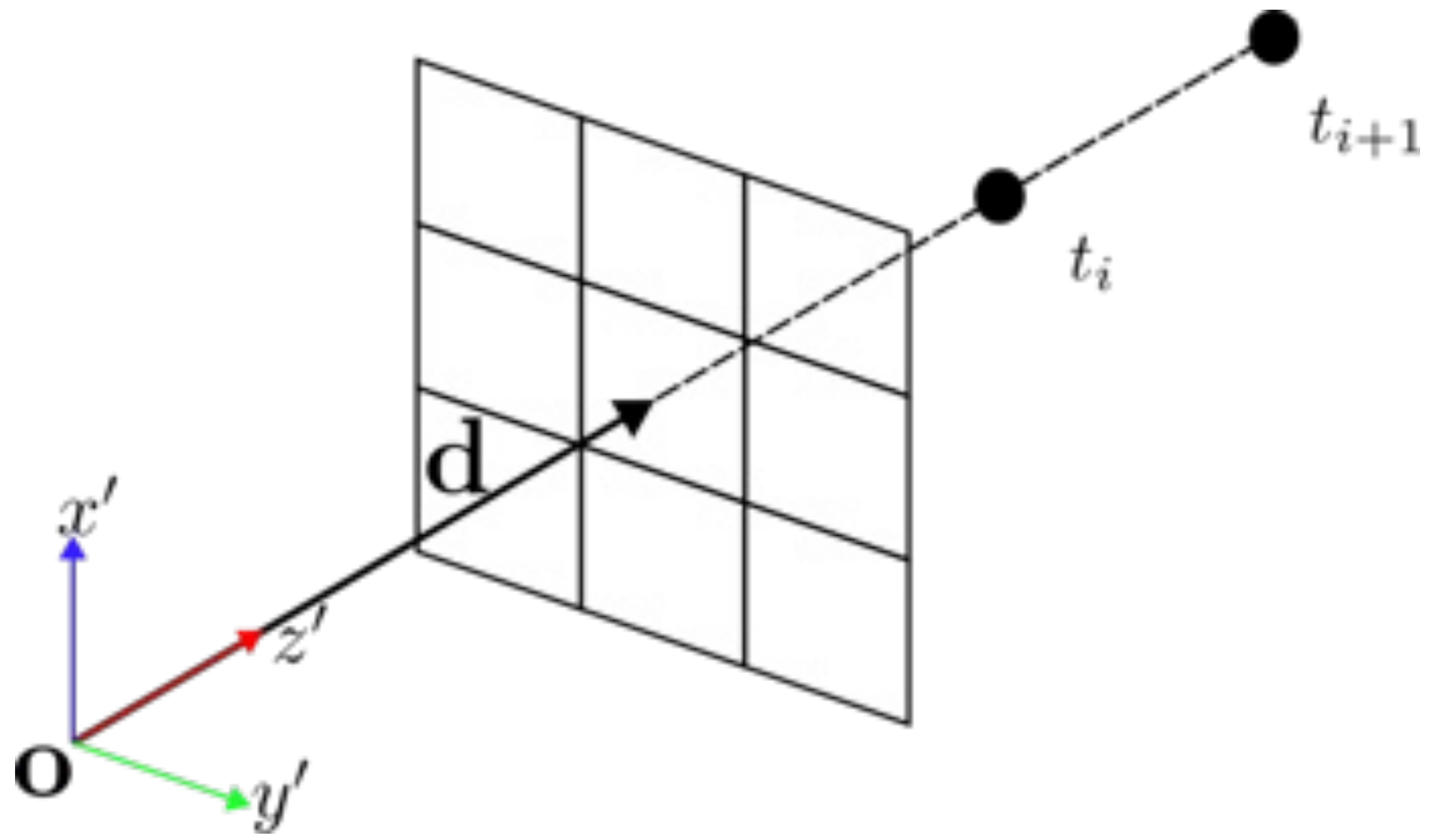
$$\gamma_I(\mathbf{d}, \mathbf{o}, \dot{\rho}, t_i, t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$

Approximated IPE

$$\gamma_I^*(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbb{E}_{\mathbf{x}\sim\mathcal{N}(\mathbf{P}\boldsymbol{\mu}, \mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^\top)}[\gamma(\mathbf{x})]$$

Motivation

NeRF: Rays and point sampling



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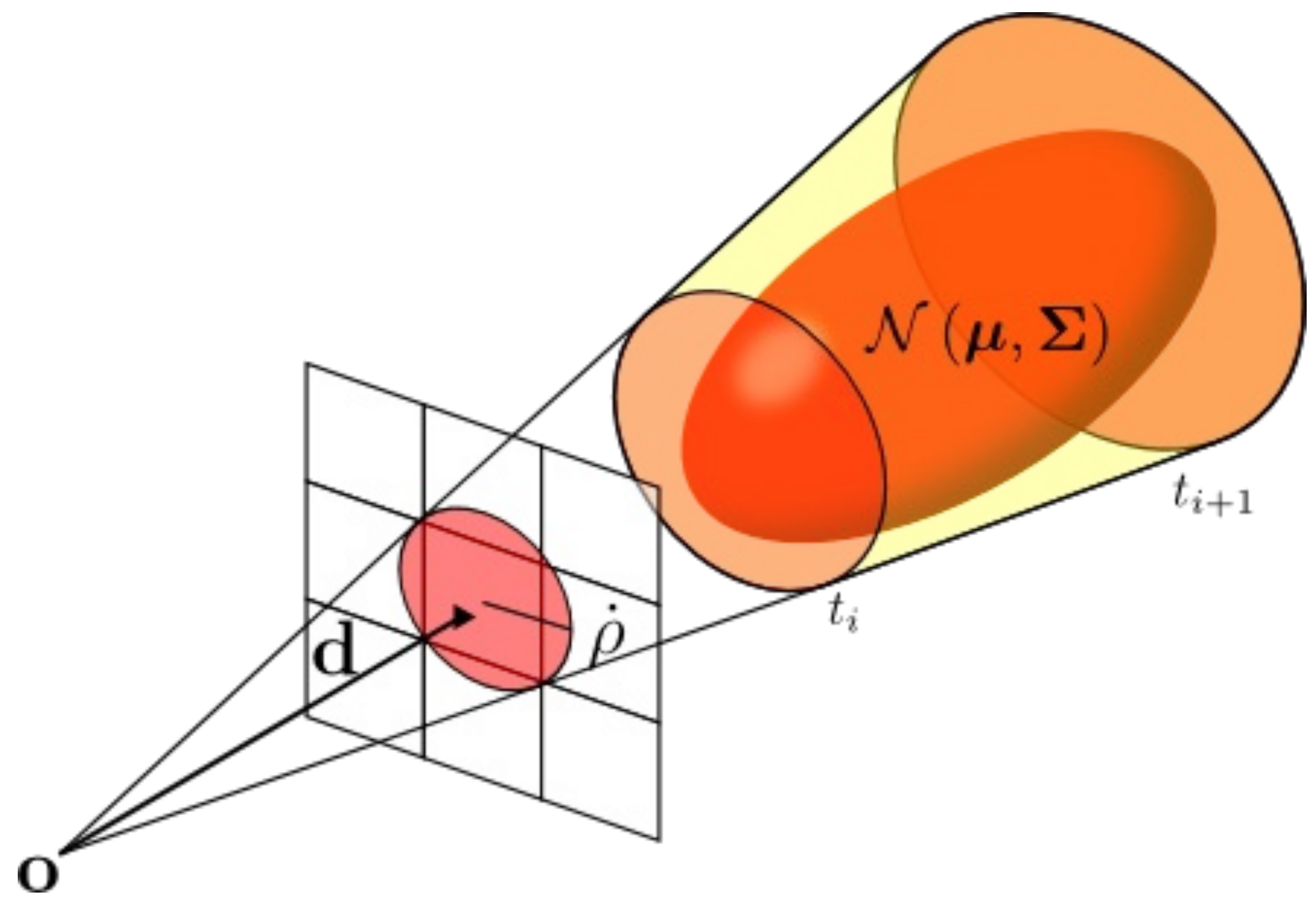


Integrated Positional Encoding

$$\gamma_I(\mathbf{d}, \mathbf{o}, \dot{\rho}, t_i, t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$

Approximated IPE

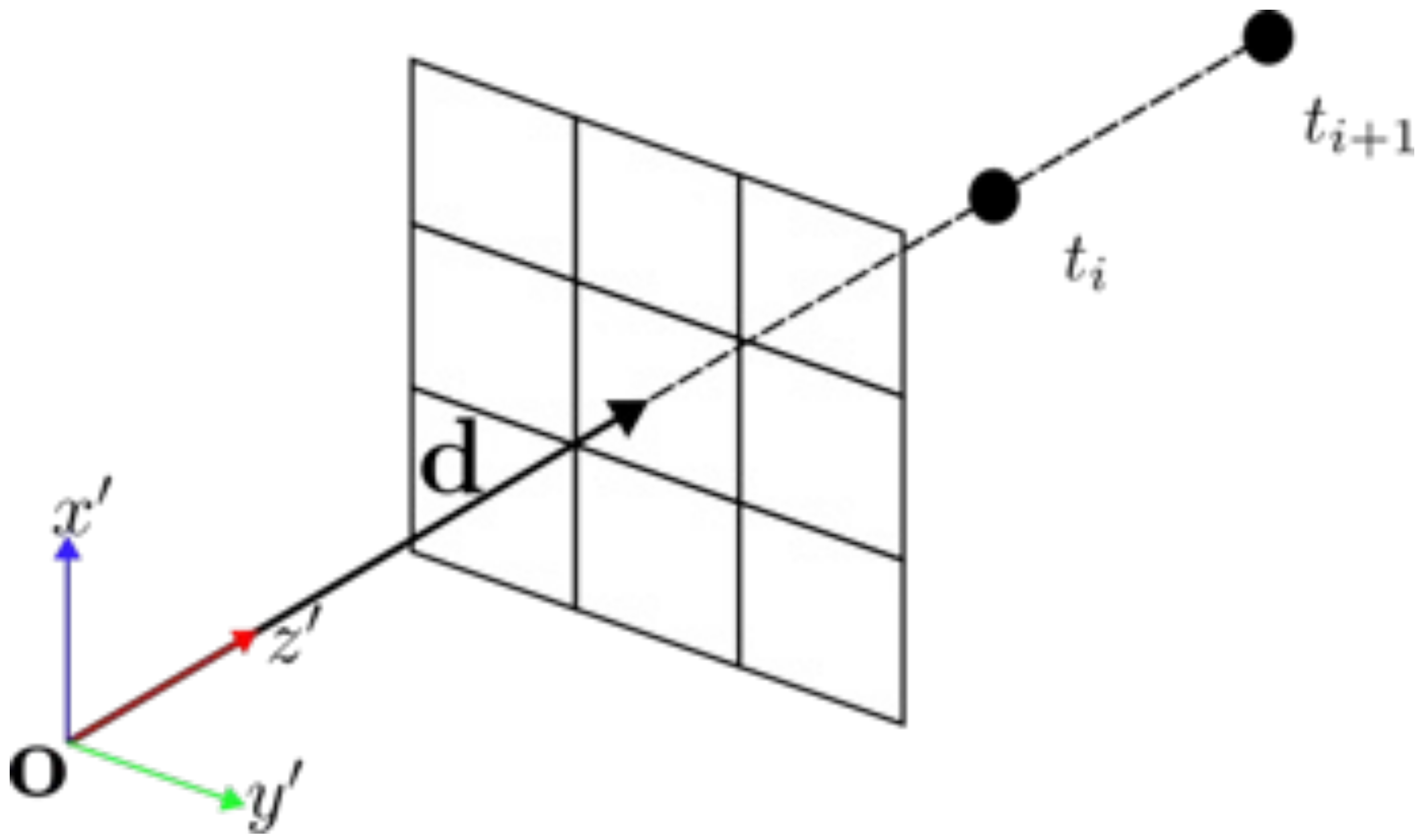
$$\gamma_I^*(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbb{E}_{\mathbf{x}\sim\mathcal{N}(\mathbf{P}\boldsymbol{\mu}, \mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^\top)}[\gamma(\mathbf{x})]$$

Mip-NeRF sampling strategy prevents aliasing and blurring

However, the Gaussian approximation degrades for large cone frustums

Motivation

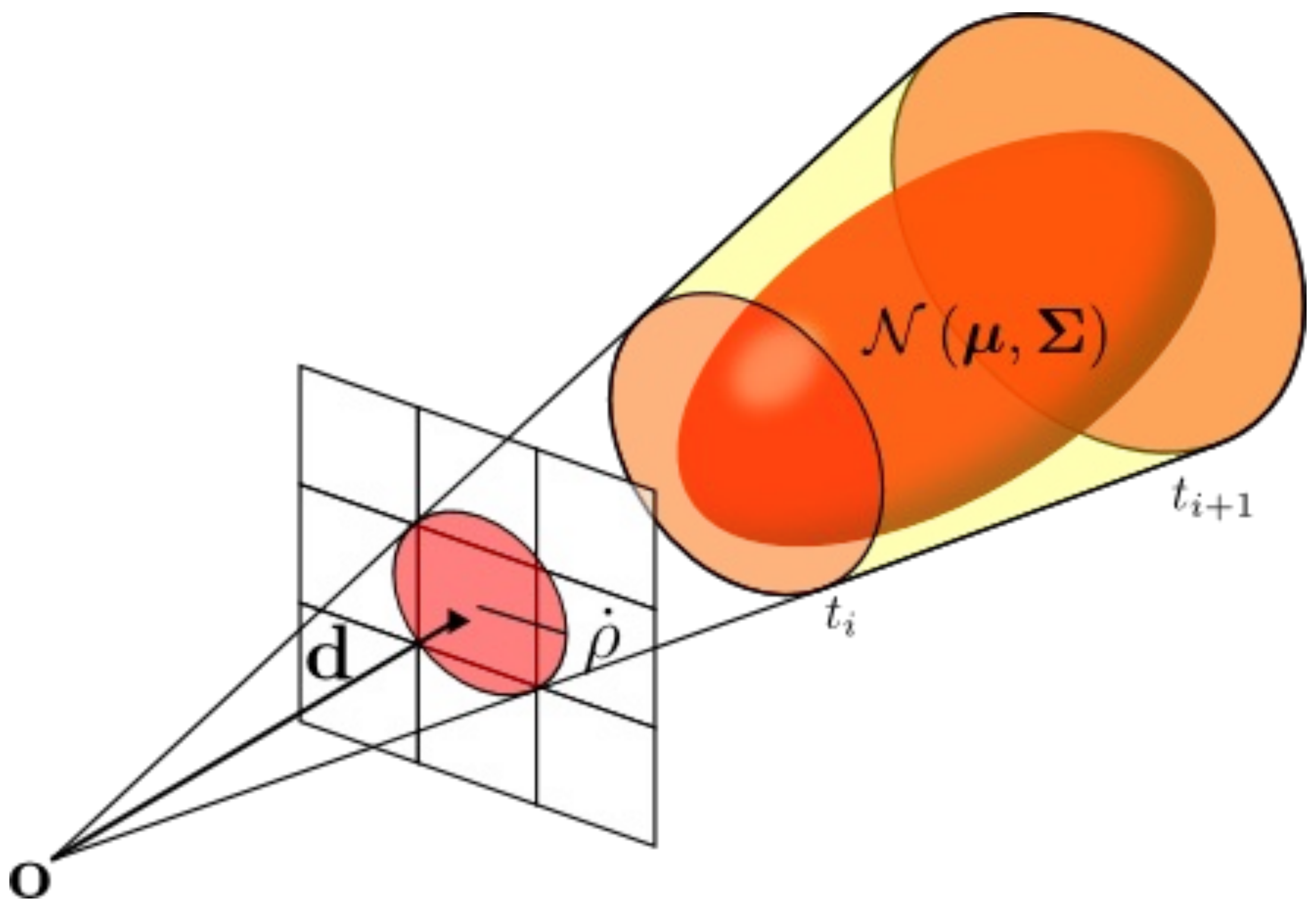
NeRF: Rays and point sampling



Positional Encoding

$$\gamma(\mathbf{x}) = [\sin(2^0\mathbf{x}), \cos(2^0\mathbf{x}), \dots, \sin(2^{L-1}\mathbf{x}), \cos(2^{L-1}\mathbf{x})]$$

Mip-NeRF: Cones and volumetric sampling



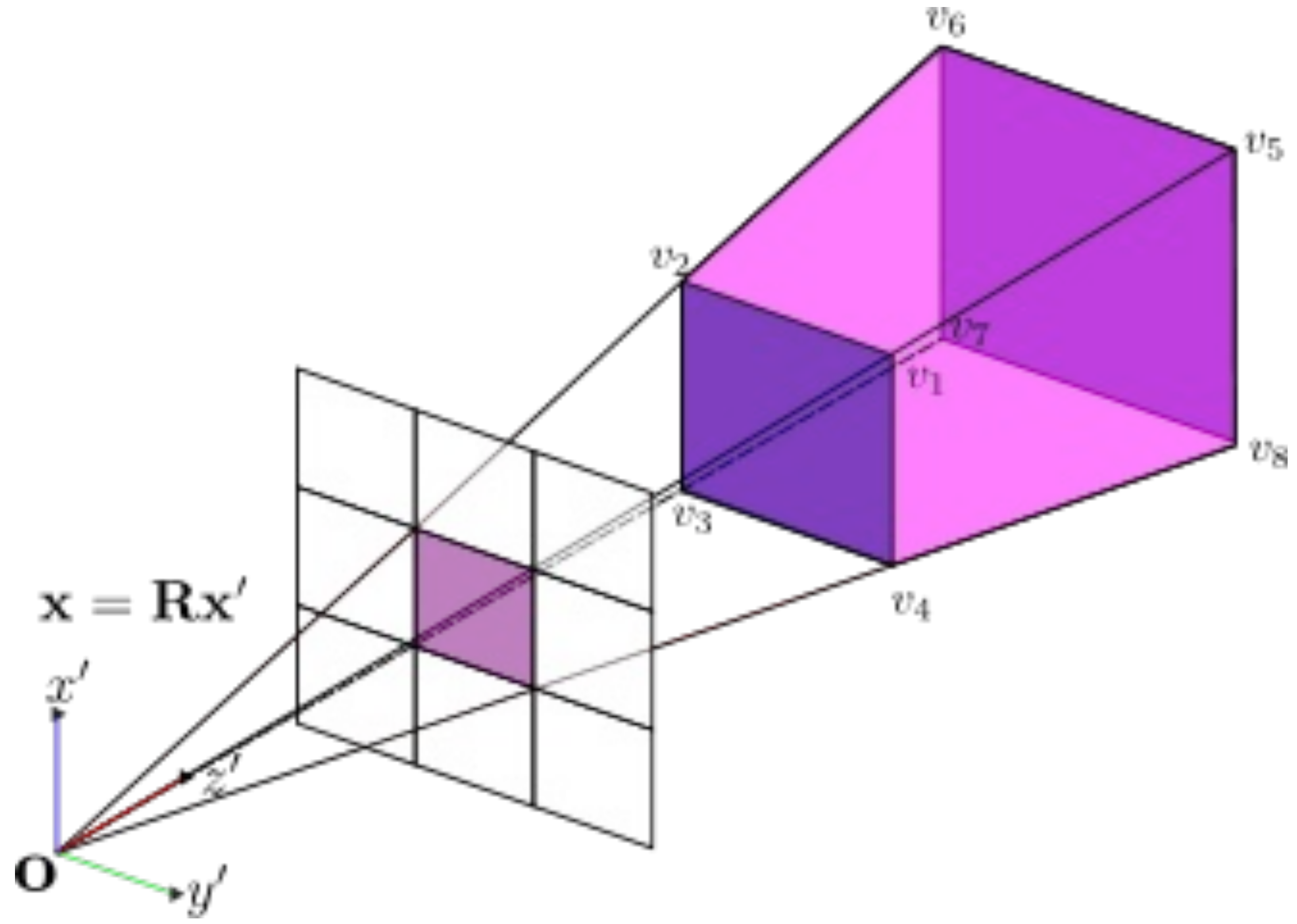
Integrated Positional Encoding

$$\gamma_I(\mathbf{d}, \mathbf{o}, \dot{\rho}, t_i, t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$

Approximated IPE

$$\gamma_I^*(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbb{E}_{\mathbf{x}\sim\mathcal{N}(\mathbf{P}\boldsymbol{\mu}, \mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^\top)}[\gamma(\mathbf{x})]$$

Exact-NeRF: Pyramids and (exact) volumetric sampling



Integrated Positional Encoding

$$\gamma_I(\mathbf{d}, \mathbf{o}, \dot{\rho}, t_i, t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$???

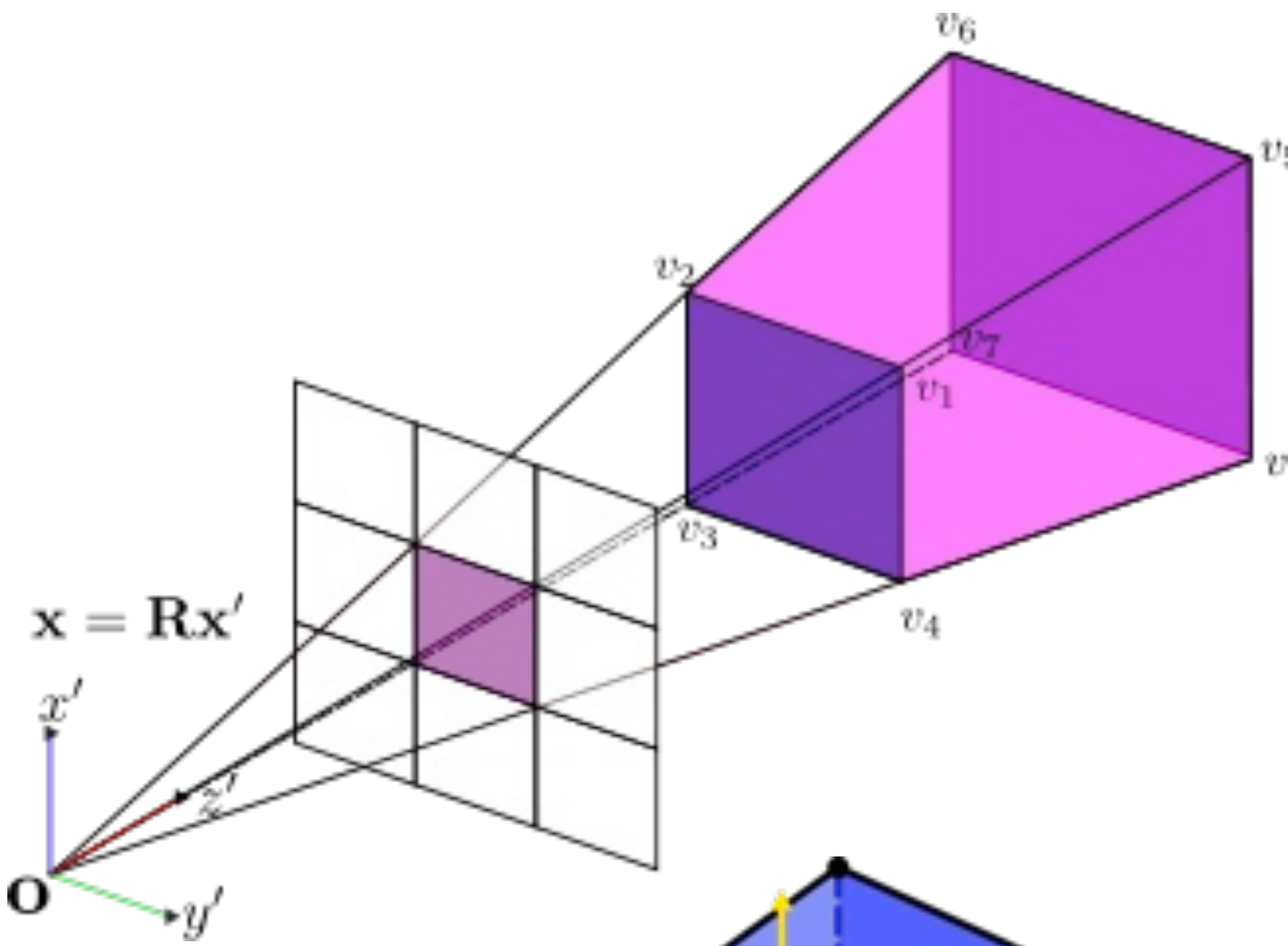
Exact-NeRF

Exact Integrated Positional Encoding

$$\gamma_I(\mathbf{d}, \mathbf{o}, \dot{\rho}, t_i, t_{i+1}) = \frac{\iiint_F \gamma(\mathbf{x})dV}{\iiint_F dV}$$

Divergence Theorem

$$\iiint \nabla \cdot \mathbf{F} dV = \oint\!\!\!\oint_{\partial S} \mathbf{F} \cdot d\mathbf{S}$$



Denominator (Volume)

$$\mathbf{F} = \frac{1}{3}[x, y, z]^\top$$

$$\iiint_F dV = \frac{1}{3}\oint\!\!\!\oint_{\partial S}[x, y, z]^\top \cdot d\mathbf{S}$$

$$\iiint_F dV = \frac{1}{6}\sum_{\tau \in \mathcal{T}} \mathbf{P}_{\tau,0} \cdot \mathbf{N}_\tau$$

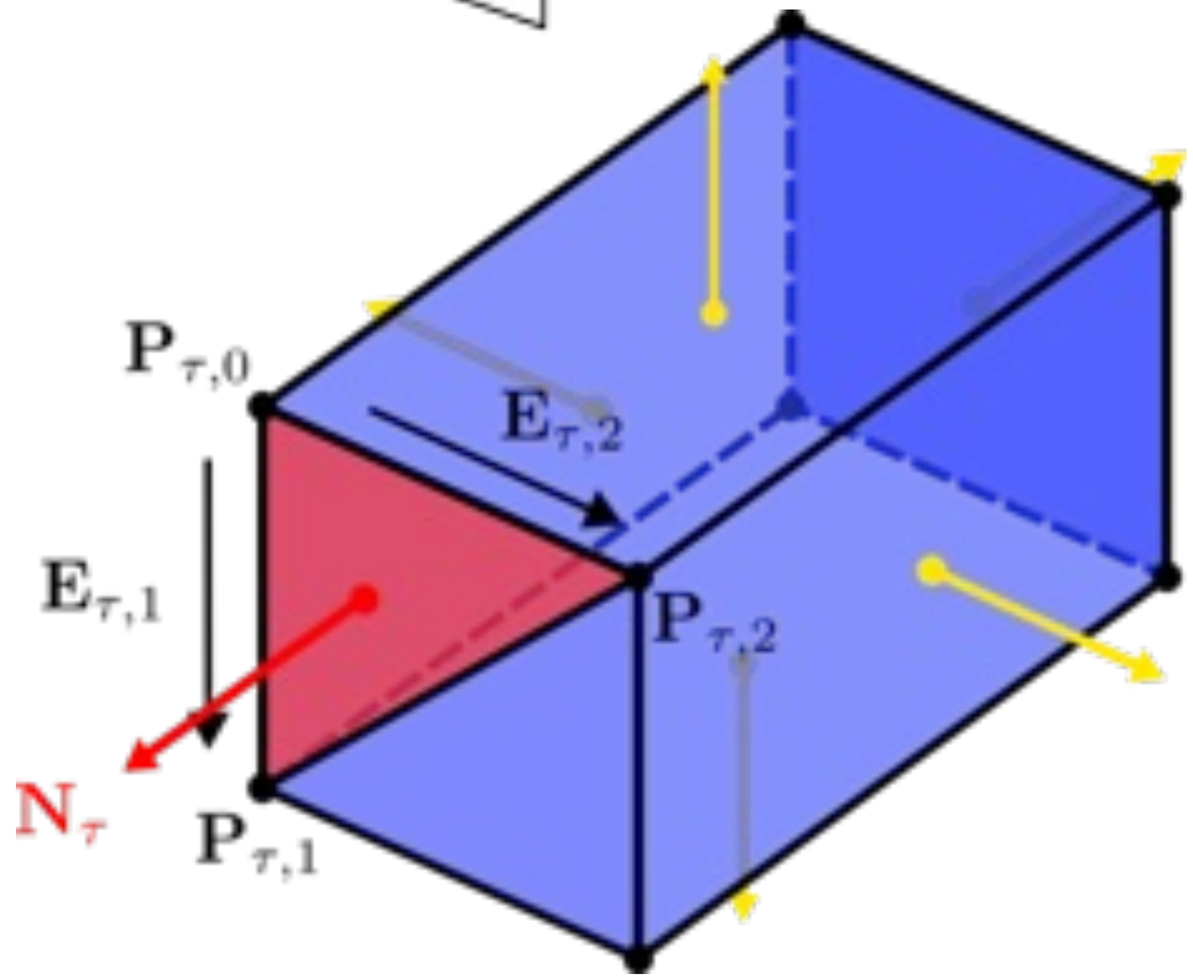
Numerator

$$\mathbf{F} = \left[-\frac{1}{2^l}\cos(2^l x), 0, 0\right]^\top$$

$$\iiint_F \sin(2^l x) dV = \oint\!\!\!\oint_{\partial S}\left[-\frac{1}{2^l}\cos(2^l x), 0, 0\right]^\top \cdot d\mathbf{S}$$

$$\iiint_F \sin(2^l \mathbf{x}_k) dV = \frac{1}{2^{3l}}\sum_{\tau \in \mathcal{T}} \sigma_{k,\tau} \mathbf{N}_\tau \cdot \mathbf{e}_k$$

$$\sigma_{k,\tau} = \frac{det\left([\mathbf{1}, \mathbf{X}_\tau^\top \mathbf{e}_k, \cos(2^l \mathbf{X}_\tau^\top \mathbf{e}_k)]\right)}{det([\mathbf{1}, \mathbf{X}_\tau^\top \mathbf{e}_k, (2^l \mathbf{X}_\tau^\top \mathbf{e}_k)^{\circ 2}])}$$



Exact-NeRF vs Mip-NeRF

Model	PSNR ↑	SSIM ↑	LPIPS ↓	DISTS ↓	Avg ↓
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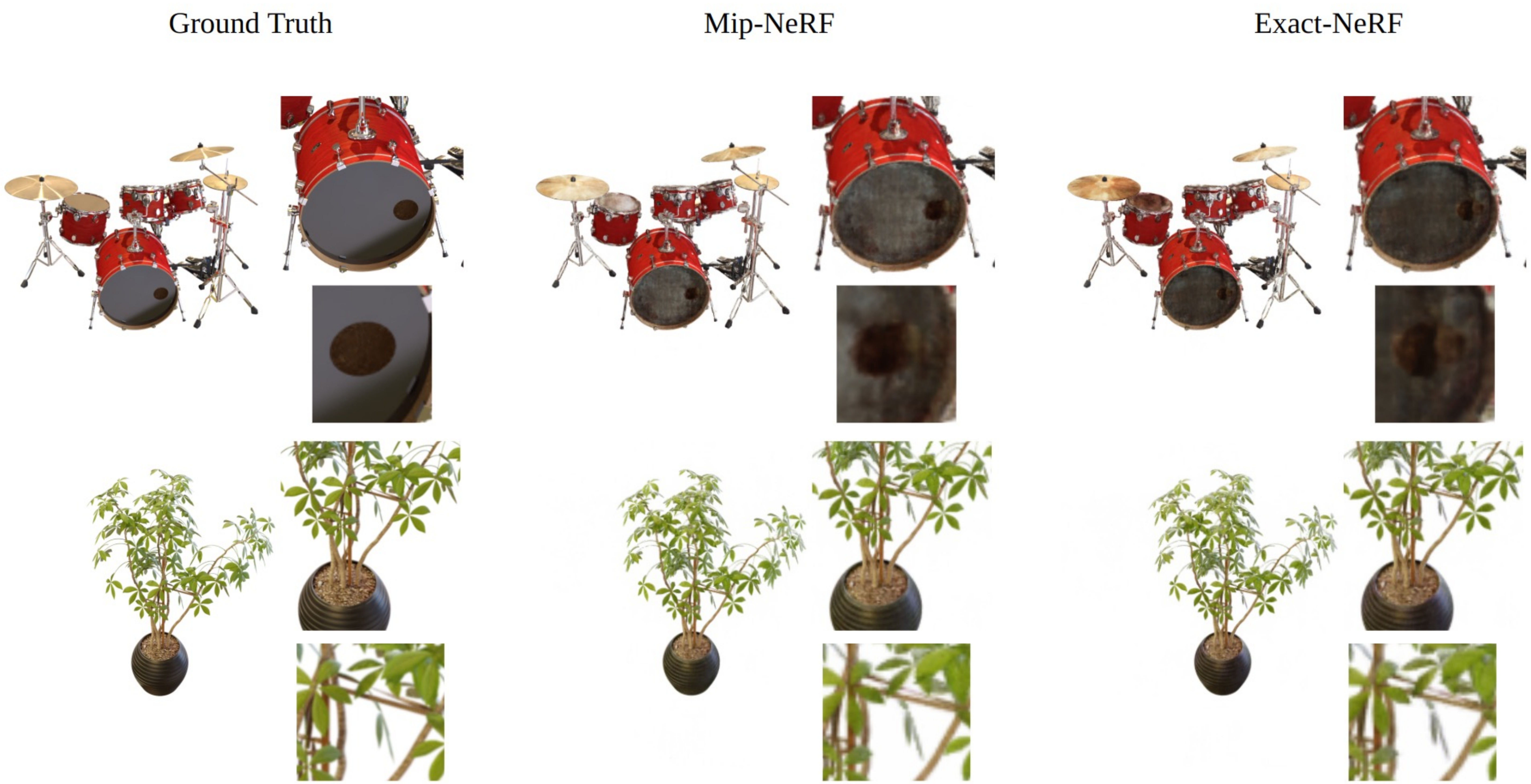

Exact-NeRF vs Mip-NeRF 360

Model	PSNR ↑	SSIM ↑	LPIPS ↓	DISTS ↓	Avg ↓
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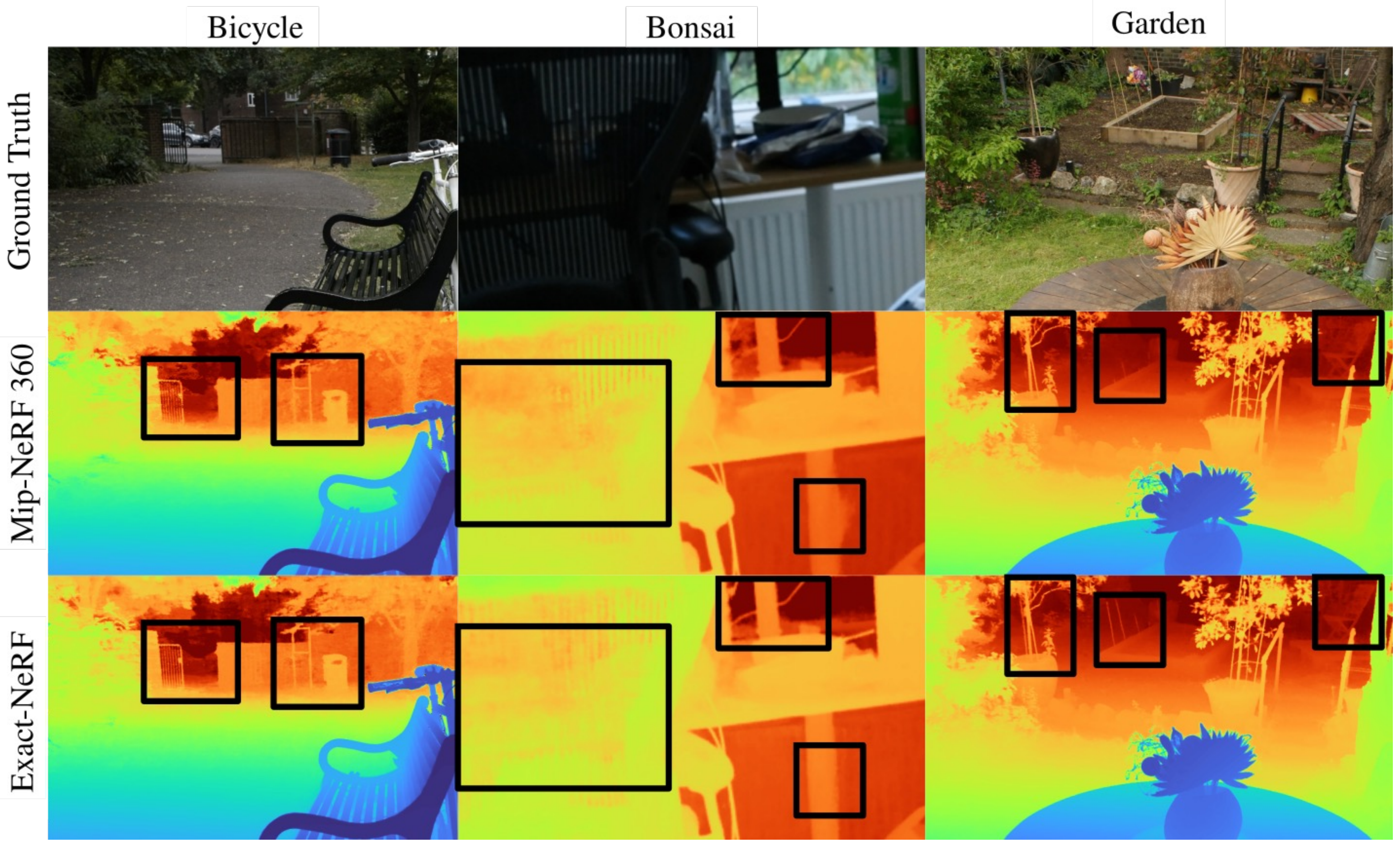

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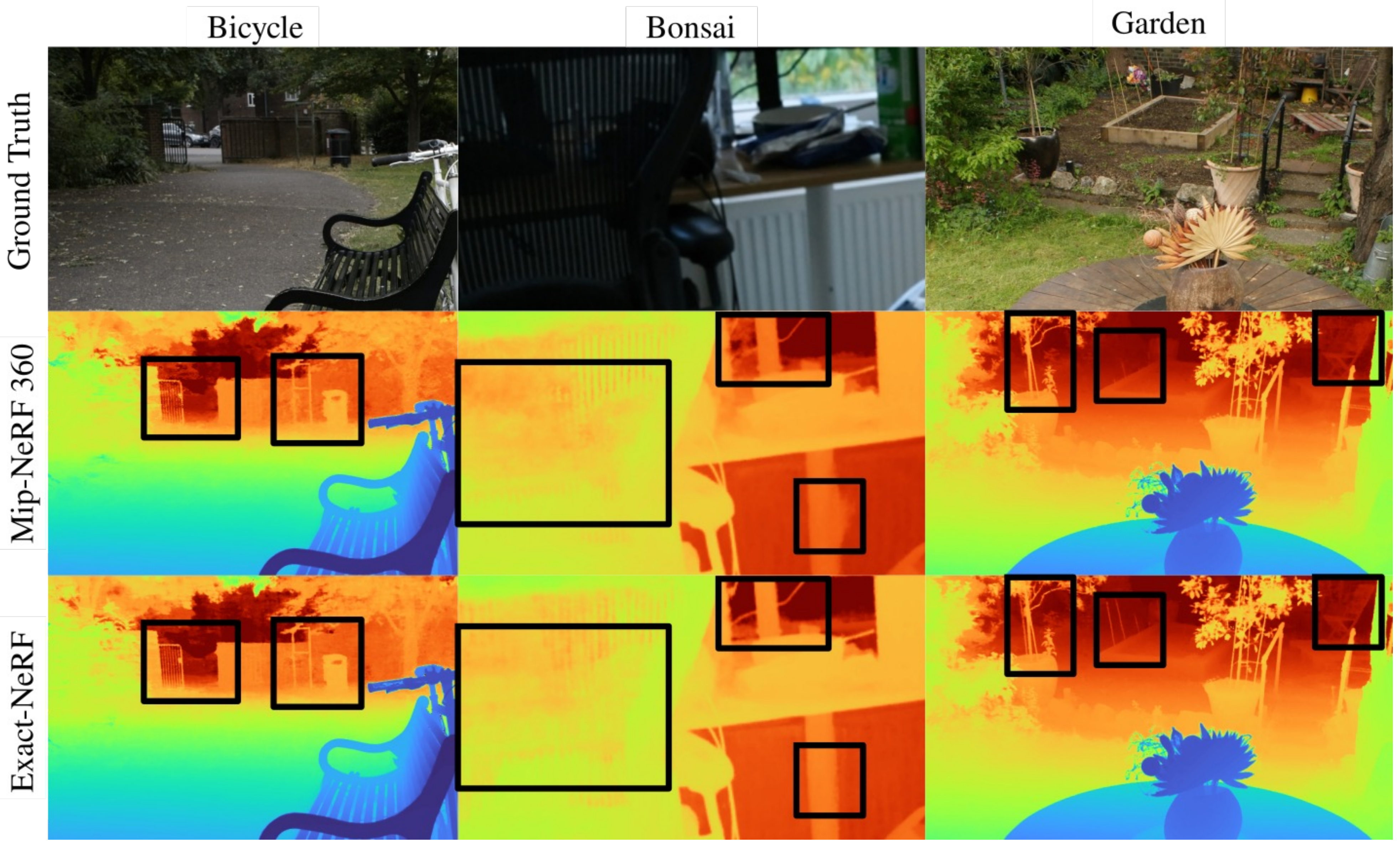
Exact-NeRF vs Mip-NeRF 360



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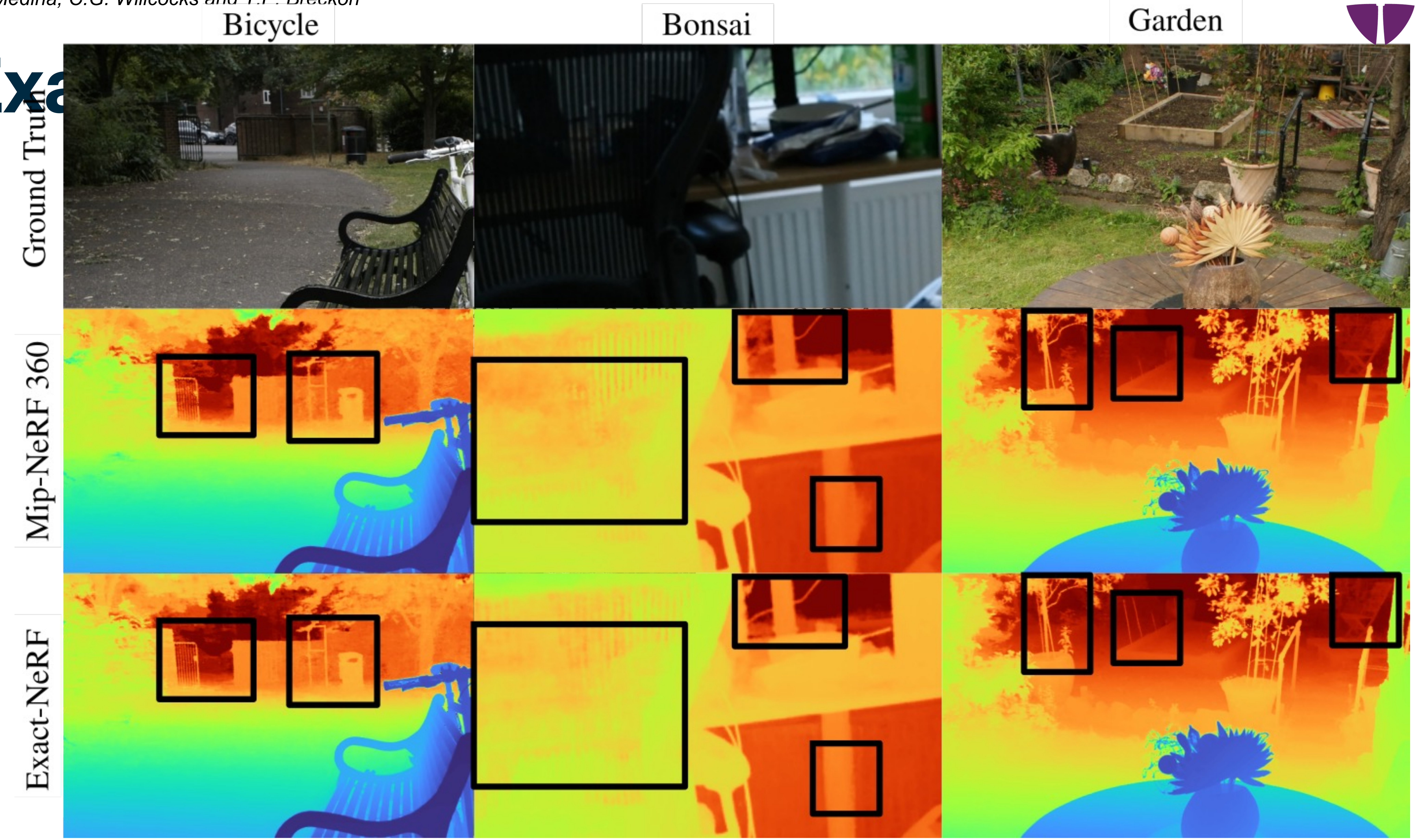

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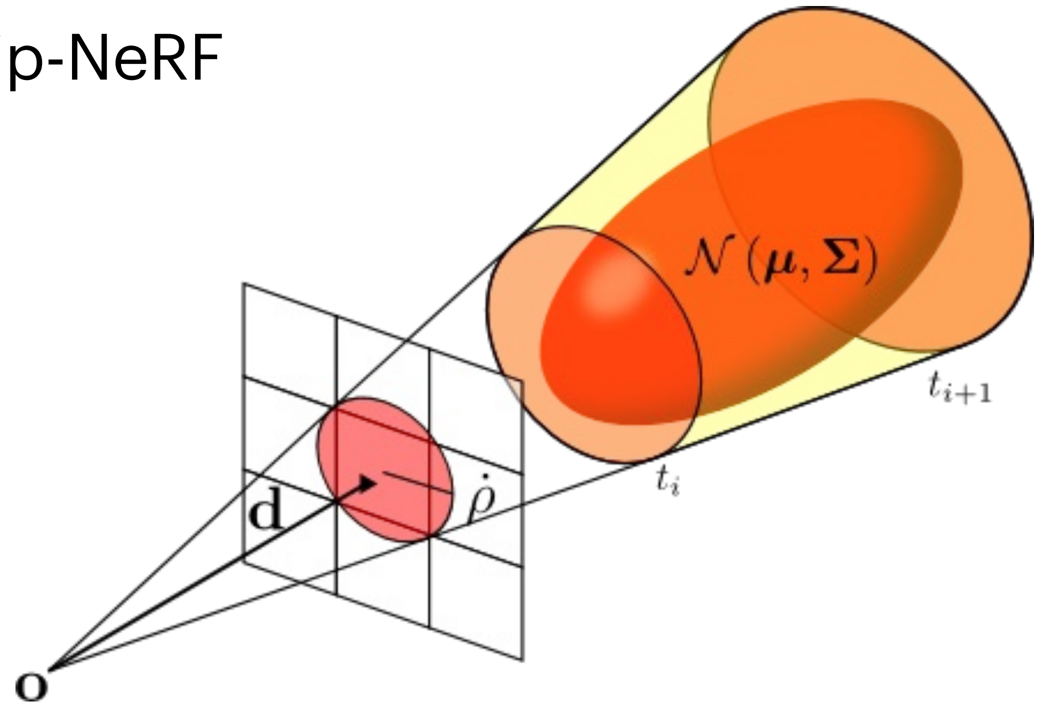
Exact-NeRF: Numerical Underflow

Our solution is prone to numerical underflow. This can be mitigated using *l'Hopital's* rule.

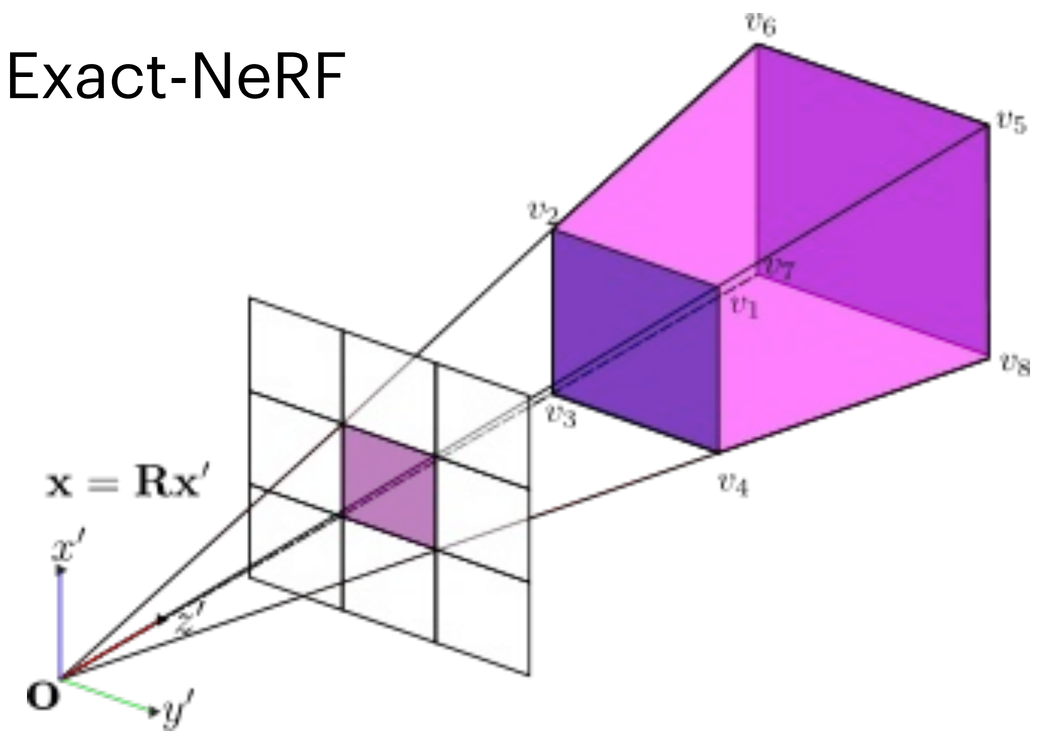
Conclusions

- Exact-NeRF is an alternative exact parameterization of the volumetric positional encoding that uses pyramids instead of cones
- Our approach has a similar performance compared to mip-NeRF and it can be implemented right off the shelf in mip-NeRF 360. Exact-NeRF shows a better reconstruction of background objects.
- The key idea of Exact-NeRF can be exploited in different applications. It also enables future exploration of new positional encodings.

Mip-NeRF



Exact-NeRF



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