KAIST

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CVPR
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Parallel Diffusion Models of Operator and Image for Blind Inverse Problems

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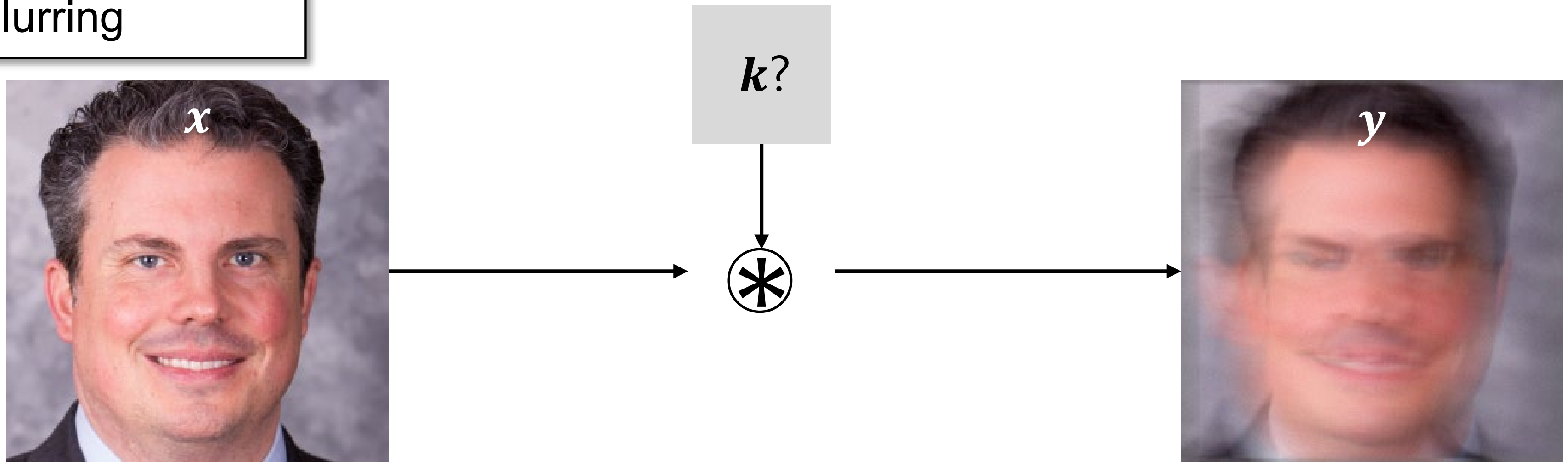
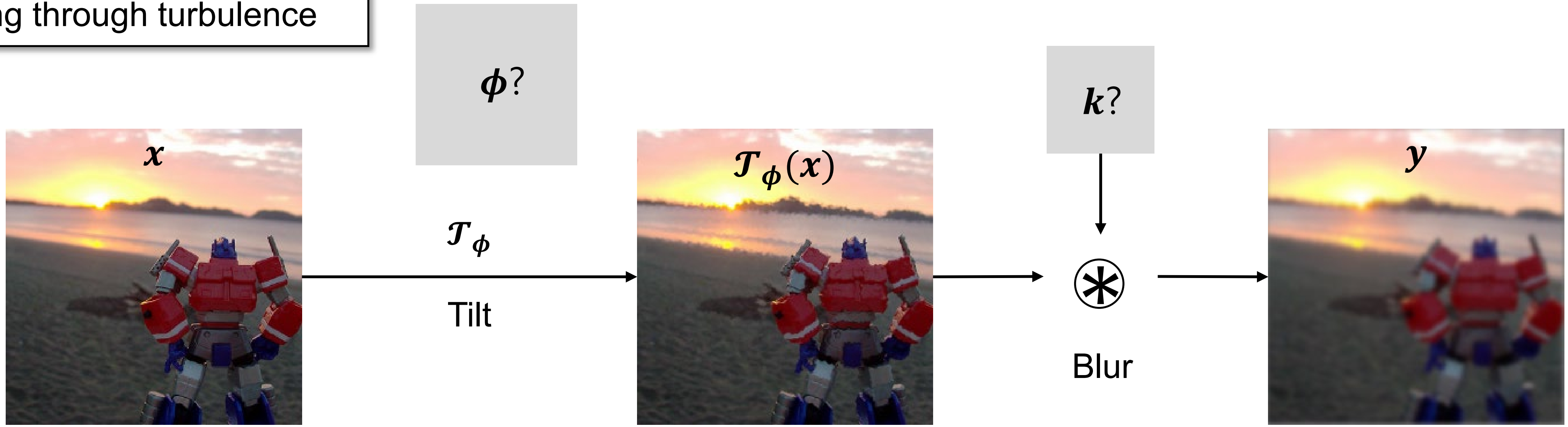
Blind inverse problems

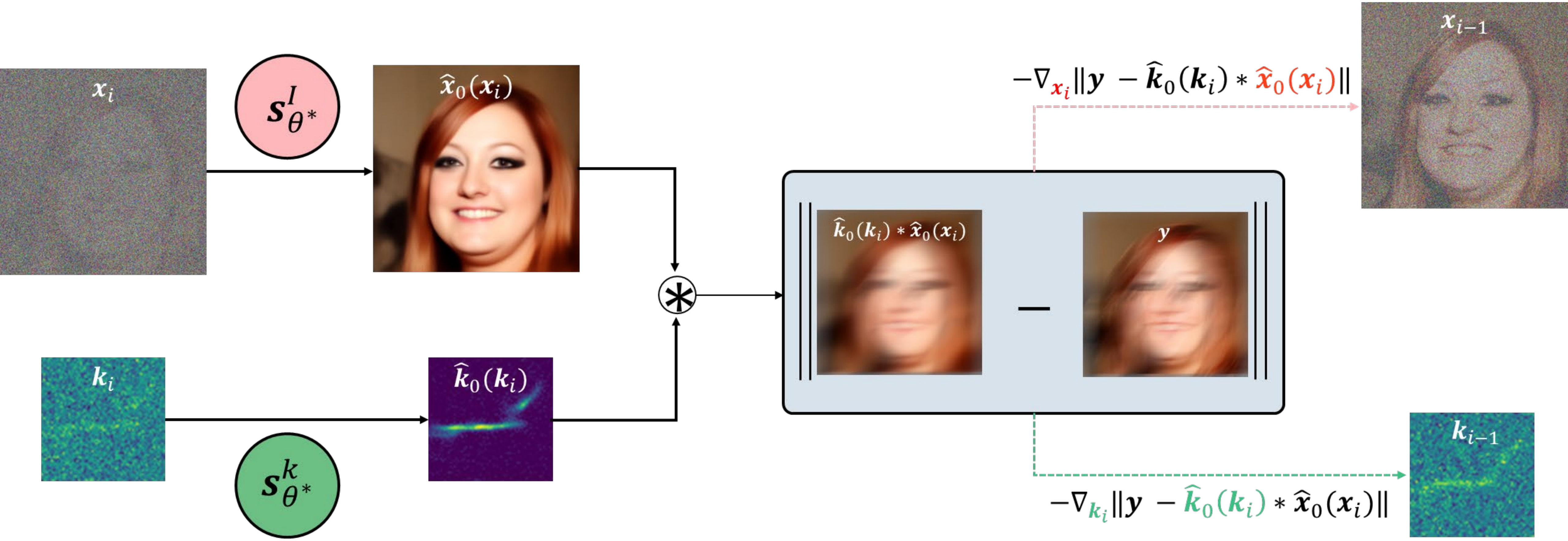
Blind deblurring



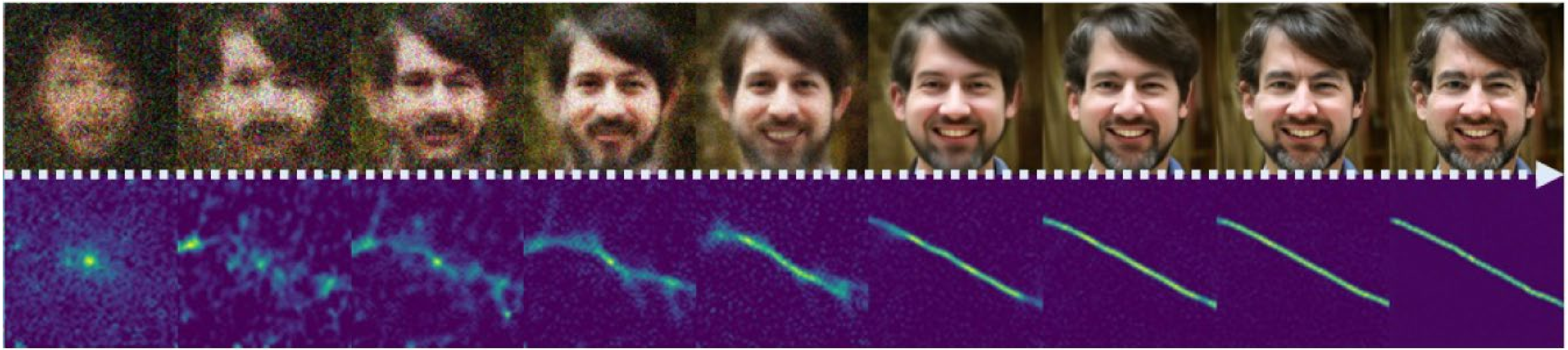
Imaging through turbulence



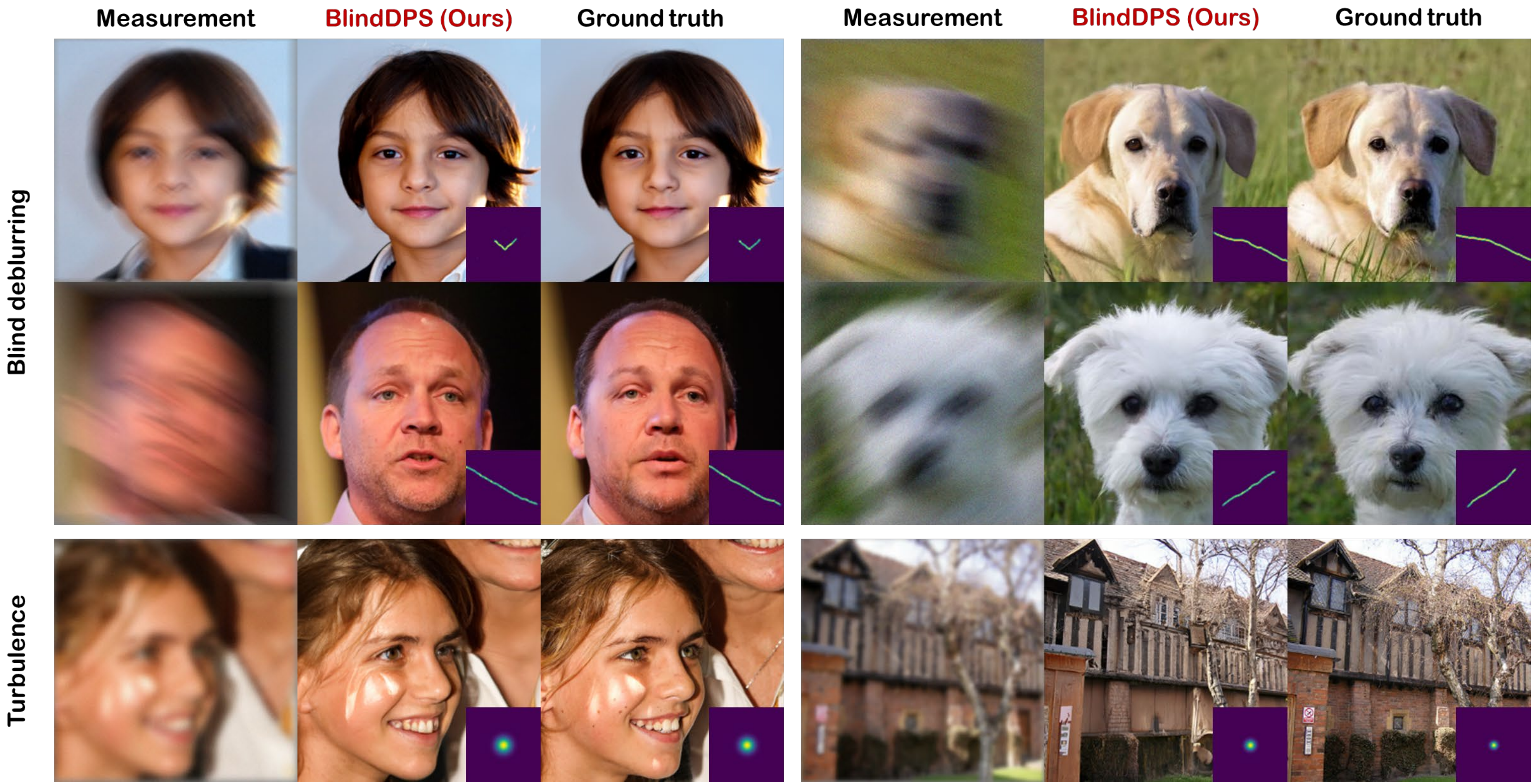
BlindDPS: parallel diffusion prior



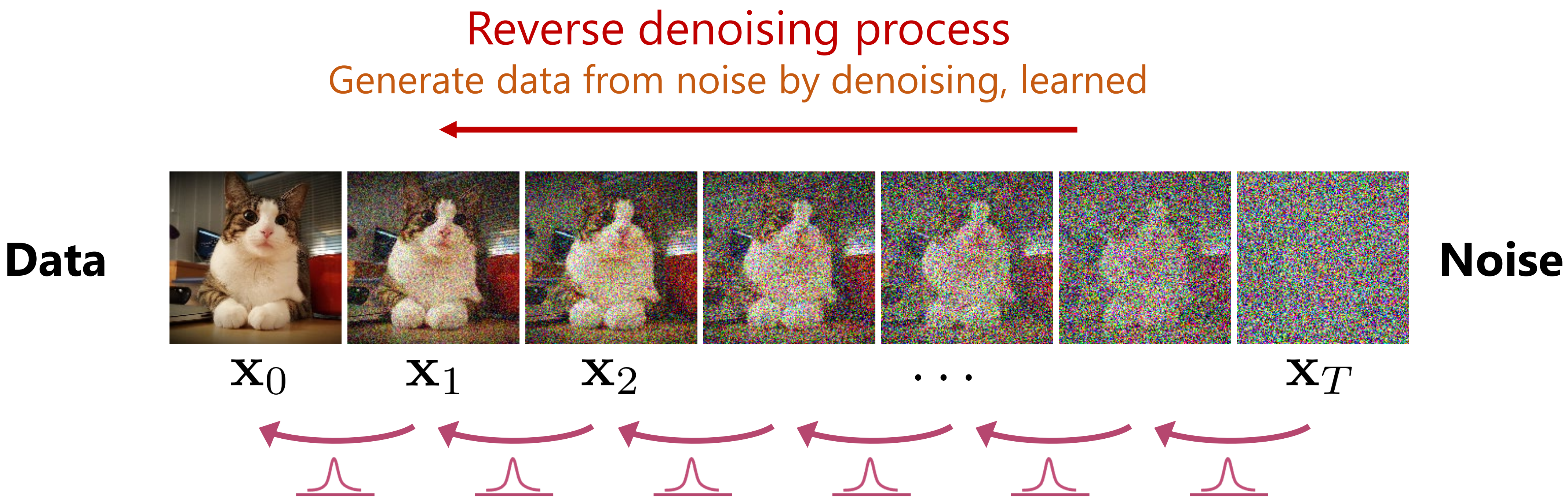
Evolution of sampling



Results

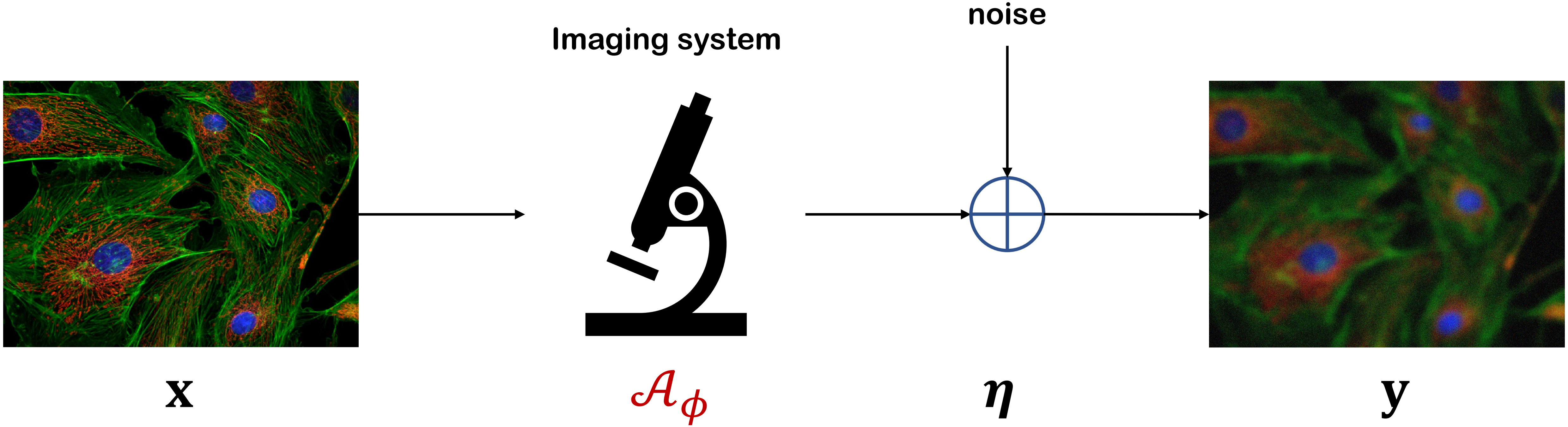

Measurement
BlindDPS (Ours)
Ground truth
Measurement
BlindDPS (Ours)
Ground truth
Blind deblurring
Turbulence

Diffusion models



$$p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mu_\theta(\mathbf{x}_t, t), r_t^2 \boldsymbol{I})$$

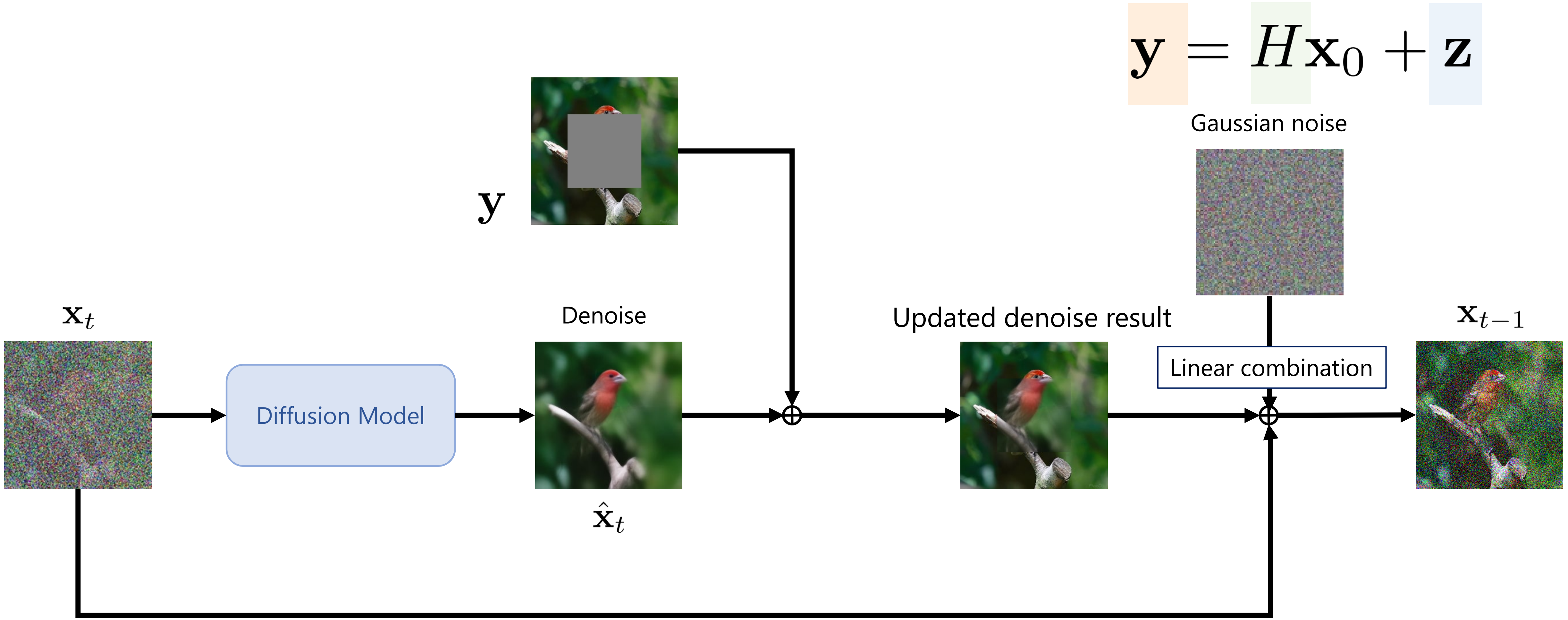
(Blind) Inverse problems



Imaging systems cast as the above **forward model**

- Acquiring the image $\mathbf{x}$: **inverse problem**
- System naturally **ill-posed**: what is the best solution?
- Often, we do not have exact information of $\mathcal{A}_\phi$

Diffusion models for inverse problems (DIS)



Wang *et al.*, "Zero-shot image restoration using denoising diffusion null-space models", ICLR 2023

Diffusion models & Bayesian inference

Reverse SDE

x_T → $d\boldsymbol{x} = [\bar{f}(\boldsymbol{x},t) - \bar{g}(t)^2 \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})]dt + \bar{g}(t)d\boldsymbol{w}$ → x_0

x_T ← $d\boldsymbol{x} = \bar{f}(\boldsymbol{x},t)dt + \bar{g}(t)d\boldsymbol{w}$ ← x_0

Forward SDE

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t)$$

Measurement process — Denoiser

Song *et al.*, “Score-based generative modeling through stochastic differential equations”, ICLR 2021

Diffusion Posterior Sampling

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t)$$

Score function: tractable

Chung *et al.*, “Diffusion Posterior Sampling for General Noisy Inverse Problems”, ICLR 2023

Diffusion Posterior Sampling

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t)$$

Time-dependent likelihood: intractable

Chung *et al.*, "Diffusion Posterior Sampling for General Noisy Inverse Problems", ICLR 2023

Diffusion Posterior Sampling

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t)$$

$$\simeq s_{\theta^*}(\mathbf{x}_t, t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\hat{\mathbf{x}}_0)$$ Tweedie

Chung *et al*., "Diffusion Posterior Sampling for General Noisy Inverse Problems", ICLR 2023

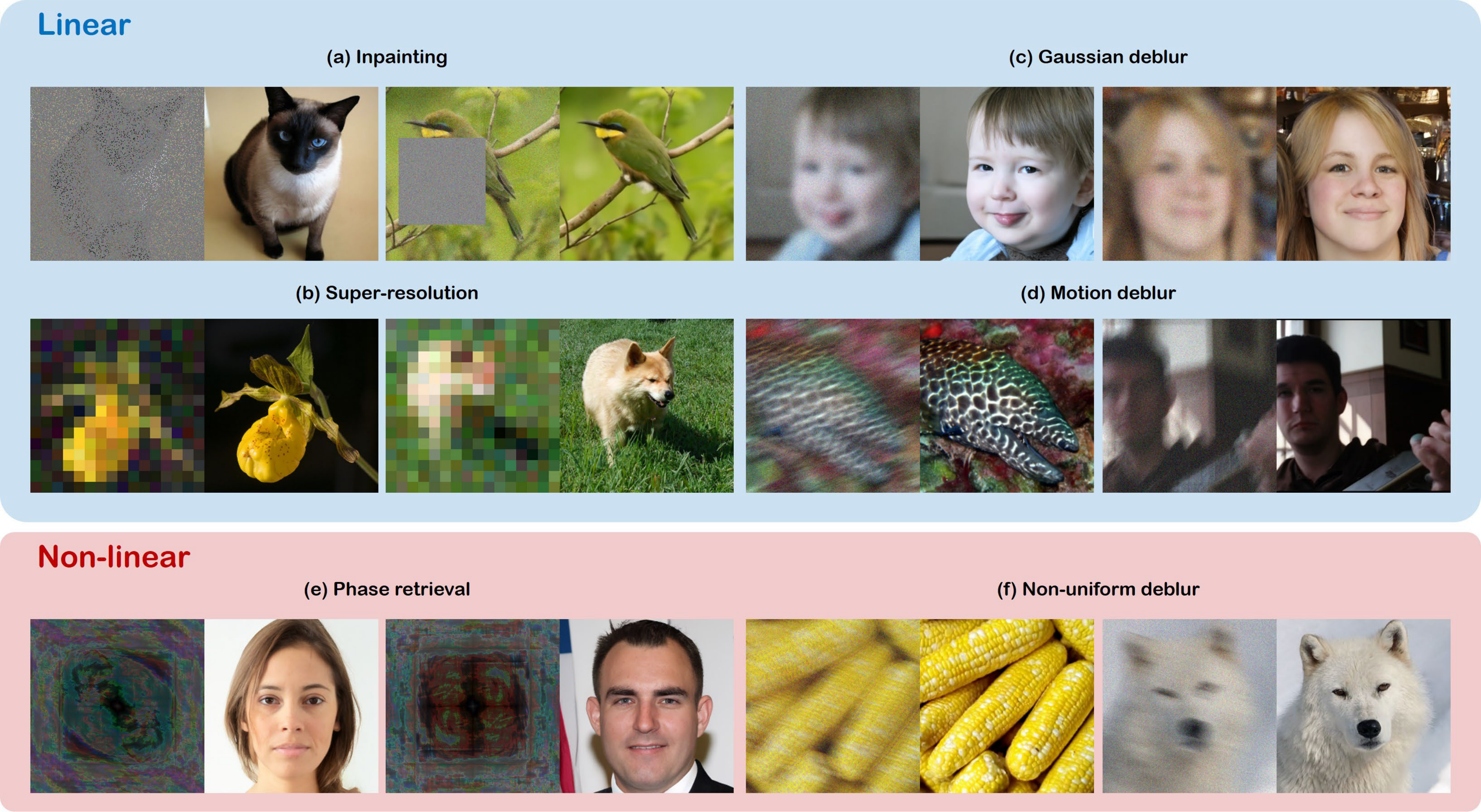
Diffusion Posterior Sampling

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t)$$
$$\simeq s_{\theta^*}(\mathbf{x}_t, t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\hat{\mathbf{x}}_0)$$

1. Gaussian $\quad \nabla_{\mathbf{x}_\mathrm{t}} \log p(\mathbf{y}|\hat{\mathbf{x}}_0) = -\rho \nabla_{\mathbf{x}_\mathrm{t}} \|\mathbf{y} - \boldsymbol{\mathcal{A}}(\hat{\mathbf{x}}_0)\|_2^2$
2. Poisson $\quad \nabla_{\mathbf{x}_\mathrm{t}} \log p(\mathbf{y}|\hat{\mathbf{x}}_0) \simeq -\rho \nabla_{\mathbf{x}_\mathrm{t}} \|\mathbf{y} - \boldsymbol{\mathcal{A}}(\hat{\mathbf{x}}_0)\|_\Lambda^2$

Chung *et al*., “Diffusion Posterior Sampling for General Noisy Inverse Problems”, ICLR 2023

Diffusion Posterior Sampling



Chung *et al*., “Diffusion Posterior Sampling for General Noisy Inverse Problems”, ICLR 2023

Diffusion Posterior Sampling

$$\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\mathbf{x}_t)$$
$$\simeq s_{\theta^*}(\mathbf{x}_t, t) + \nabla_{\mathbf{x}_t} \log p(\mathbf{y}|\hat{\mathbf{x}}_0)$$

1. Gaussian $\quad \nabla_{\mathbf{x}_\mathrm{t}} \log p(\mathbf{y}|\hat{\mathbf{x}}_0) = -\rho \nabla_{\mathbf{x}_\mathrm{t}} \|\mathbf{y} - \boldsymbol{\mathcal{A}}(\hat{\mathbf{x}}_0)\|_2^2$

2. Poisson $\quad \nabla_{\mathbf{x}_\mathrm{t}} \log p(\mathbf{y}|\hat{\mathbf{x}}_0) \simeq -\rho \nabla_{\mathbf{x}_\mathrm{t}} \|\mathbf{y} - \boldsymbol{\mathcal{A}}(\hat{\mathbf{x}}_0)\|_\Lambda^2$

Assumes knowledge of the forward operator $\boldsymbol{\mathcal{A}}$

Chung *et al*., “Diffusion Posterior Sampling for General Noisy Inverse Problems”, ICLR 2023

Blind inverse problem

$$\mathbf{y} = \mathcal{A}(\mathbf{x}) + \boldsymbol{\eta}$$

unknown

Blind inverse problem

$$\mathbf{y} = \mathcal{A}_{\phi}(\mathbf{x}) + \boldsymbol{\eta}$$

unknown

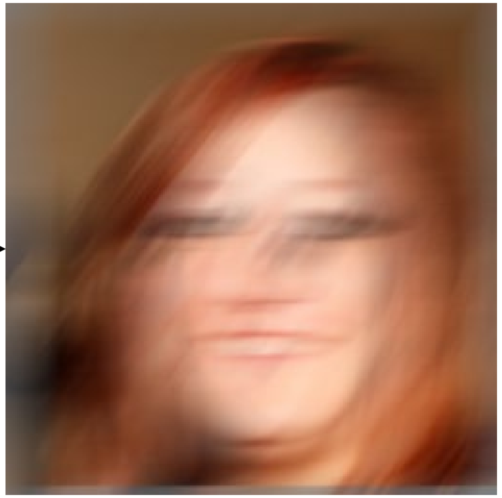
Blind inverse problem

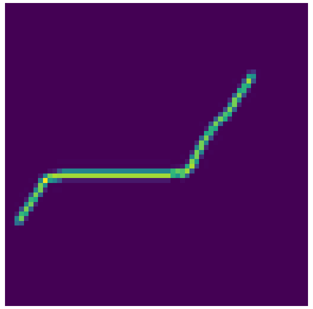
$$\mathbf{y} = \mathcal{A}_{\phi}(\mathbf{x}) + \boldsymbol{\eta}$$

unknown

$\mathcal{A}_{\phi}$

Blind deconvolution (deblurring)





Blind inverse problem

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k_0}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k_0} * \boldsymbol{x_0}, \sigma^2 \boldsymbol{I})$$

$$p(\boldsymbol{x}_0, \boldsymbol{k_0}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x_0}, \boldsymbol{k_0})p(\boldsymbol{x_0})p(\boldsymbol{k_0})$$

Blind inverse problem

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_\mathbf{0}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k}_\mathbf{0} * \boldsymbol{x}_\mathbf{0}, \sigma^2\boldsymbol{I})$$

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Image prior

Diffusion prior for image / operator

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k_0}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k_0} * \boldsymbol{x_0}, \sigma^2 \boldsymbol{I})$$

$$p(\boldsymbol{x}_0, \boldsymbol{k_0}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x_0}, \boldsymbol{k_0}) p(\boldsymbol{x_0}) p(\boldsymbol{k_0})$$

kernel prior

Diffusion prior for image / operator

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k}_0 * \boldsymbol{x}_0, \sigma^2 \boldsymbol{I})$$

$$p(\boldsymbol{x}_0, \boldsymbol{k}_0|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0) p(\boldsymbol{x}_0) p(\boldsymbol{k}_0)$$

Choices? Diffusion prior!

Diffusion prior for image / operator

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k}_0 * \boldsymbol{x}_0, \sigma^2 \boldsymbol{I})$$

$$p(\boldsymbol{x}_0, \boldsymbol{k}_0|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0)p(\boldsymbol{x}_0)p(\boldsymbol{k}_0)$$

Trained independently with **DSM**

$$\nabla_{\boldsymbol{x}} \log p(\boldsymbol{x}) \simeq s_\theta^I(\boldsymbol{x})$$

$$\nabla_{\boldsymbol{k}} \log p(\boldsymbol{k}) \simeq s_\theta^k(\boldsymbol{k})$$

BlindDPS: posterior sampling

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0) \propto p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0)p(\boldsymbol{x}_0)p(\boldsymbol{k}_0)$$

$$d\boldsymbol{x} = \left[-\frac{\beta(t)}{2}\boldsymbol{x} - \beta(t)\nabla_{\boldsymbol{x}_t}\log p(\boldsymbol{x}_t)\right]dt + \sqrt{\beta(t)}d\,\bar{\boldsymbol{w}}$$

$$d\boldsymbol{k} = \left[-\frac{\beta(t)}{2}\boldsymbol{k} - \beta(t)\nabla_{\boldsymbol{k}_t}\log p(\boldsymbol{k}_t)\right]dt + \sqrt{\beta(t)}d\,\bar{\boldsymbol{w}}$$

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BlindDPS: posterior sampling

$$p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0) \propto p(\boldsymbol{y}|\boldsymbol{x}_0, \boldsymbol{k}_0)p(\boldsymbol{x}_0)p(\boldsymbol{k}_0)$$

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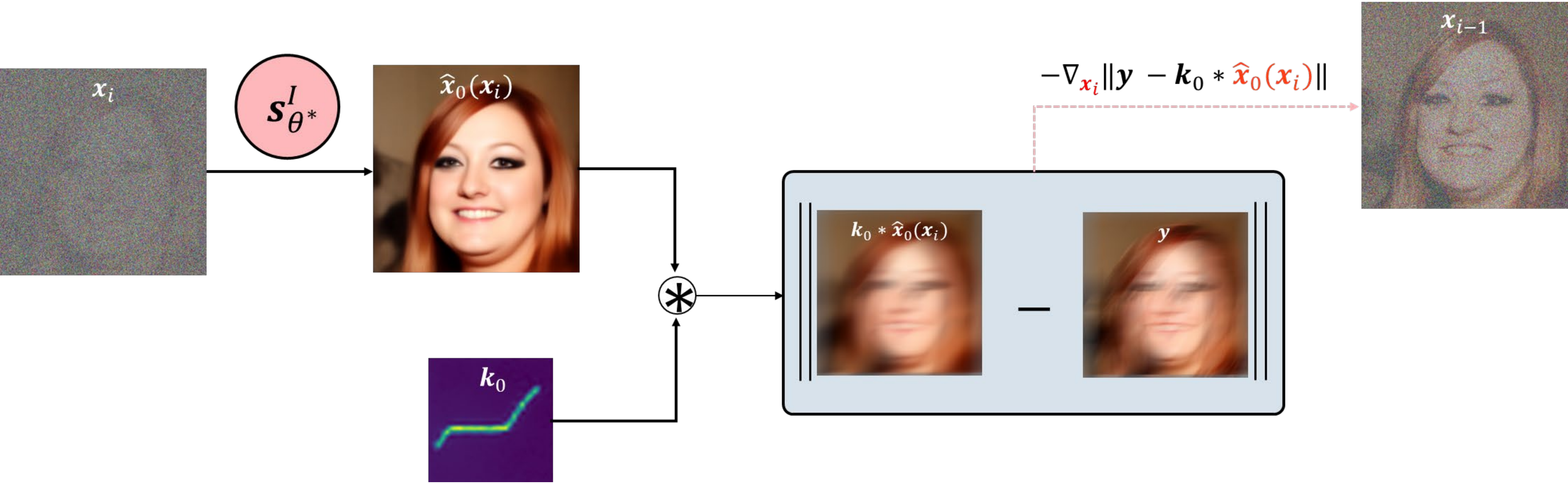
Theorem. With similar approximation gap as in DPS,

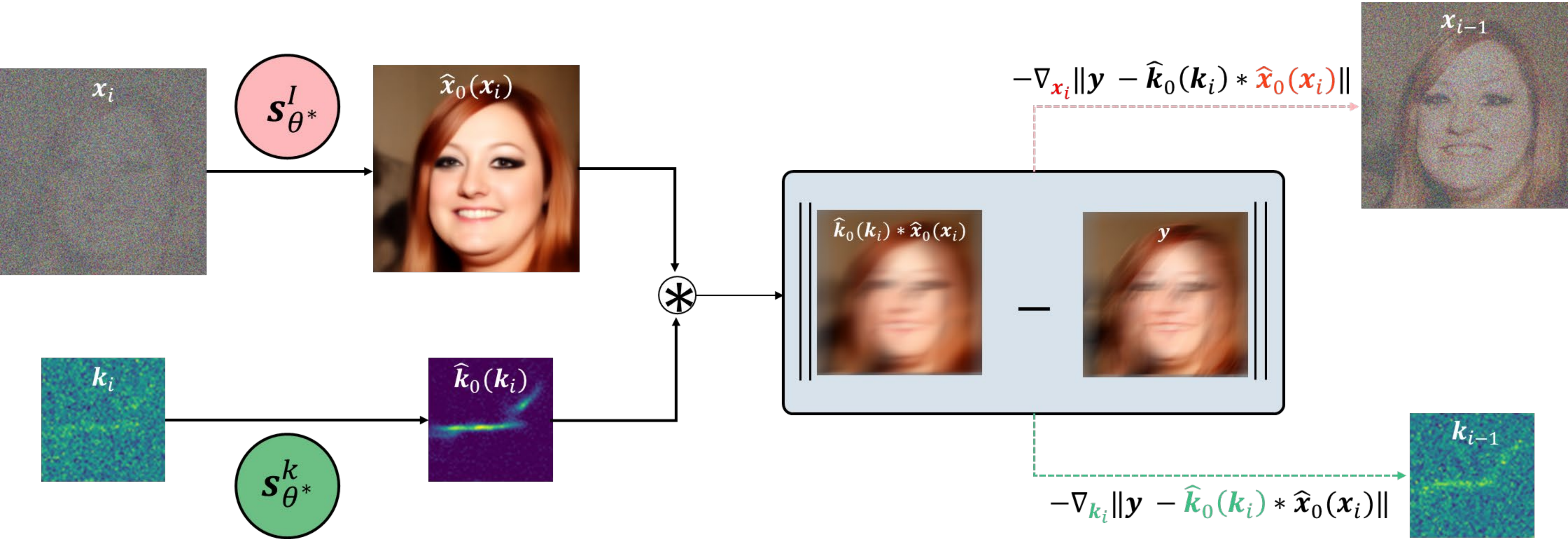
$$\nabla_{\boldsymbol{x}_t} \log p(\boldsymbol{y}|\boldsymbol{x}_t, \boldsymbol{k}_t) \simeq \nabla_{\mathbf{x}_t} \log p\left(\boldsymbol{y}|\hat{\boldsymbol{x}}_0(\boldsymbol{x}_t), \hat{\boldsymbol{k}}_0(\boldsymbol{k}_t)\right)$$

$$\nabla_{\boldsymbol{k}_t} \log p(\boldsymbol{y}|\boldsymbol{x}_t, \boldsymbol{k}_t) \simeq \nabla_{\boldsymbol{k}_t} \log p\left(\boldsymbol{y}|\hat{\boldsymbol{x}}_0(\boldsymbol{x}_t), \hat{\boldsymbol{k}}_0(\boldsymbol{k}_t)\right)$$

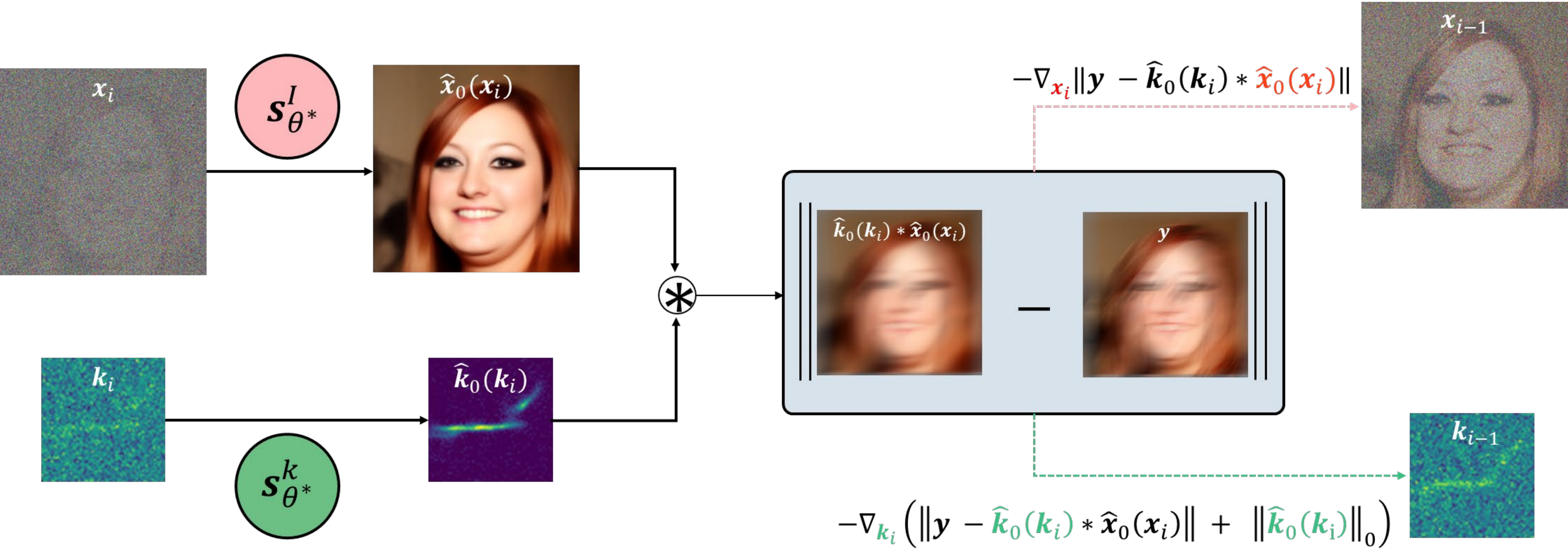
Single diffusion for non-blind



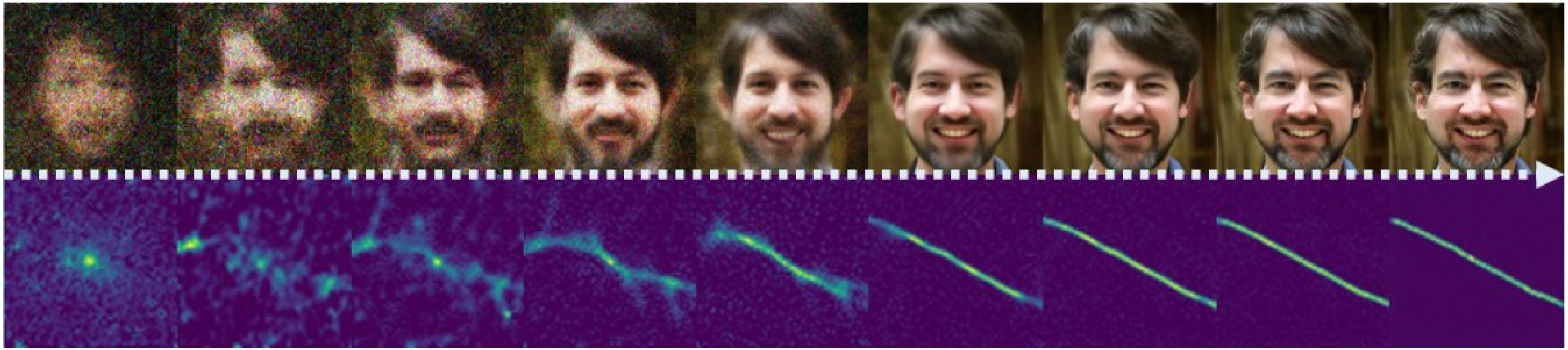
Parallel diffusion for blind



Parallel diffusion for blind

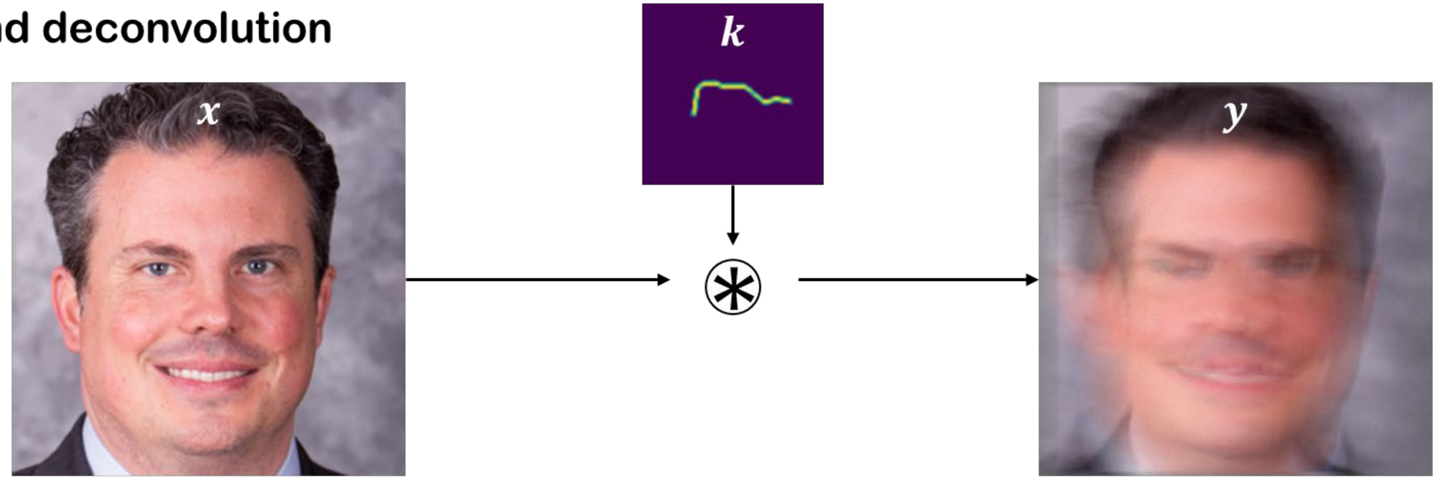


Evolution of sampling

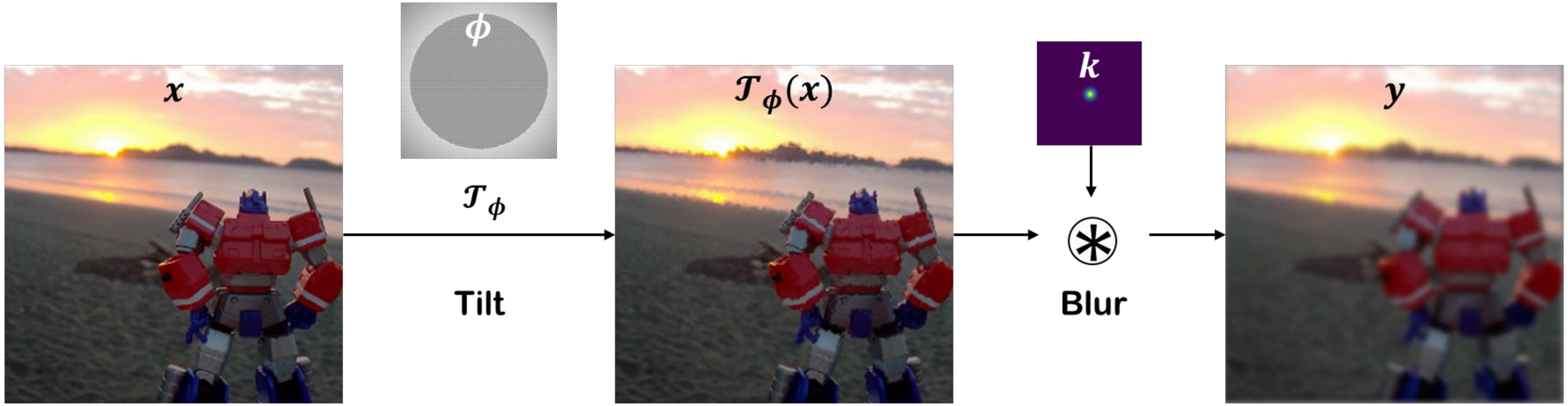


General framework for Blind IP

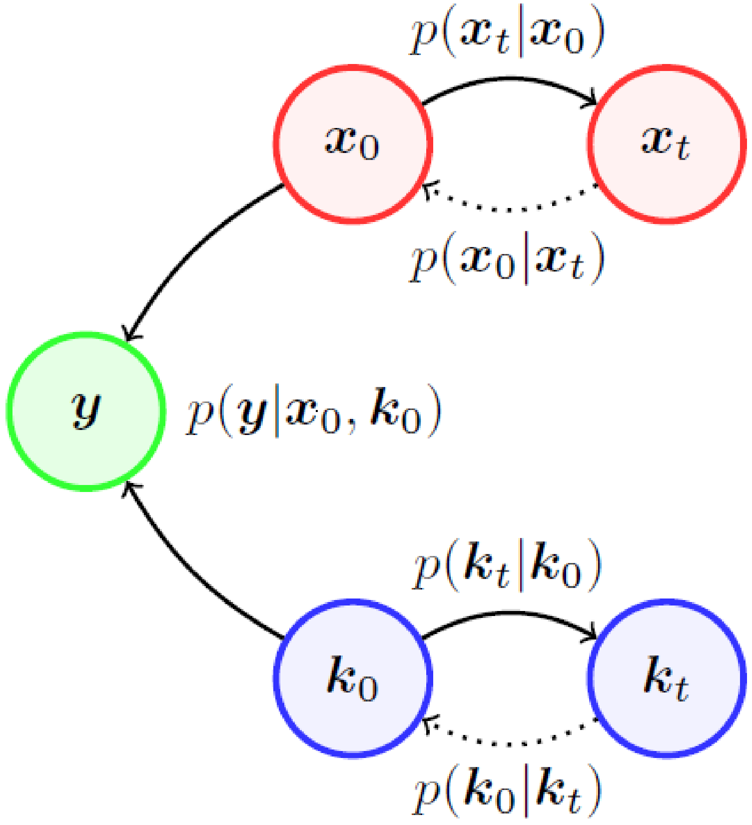
(a) Blind deconvolution



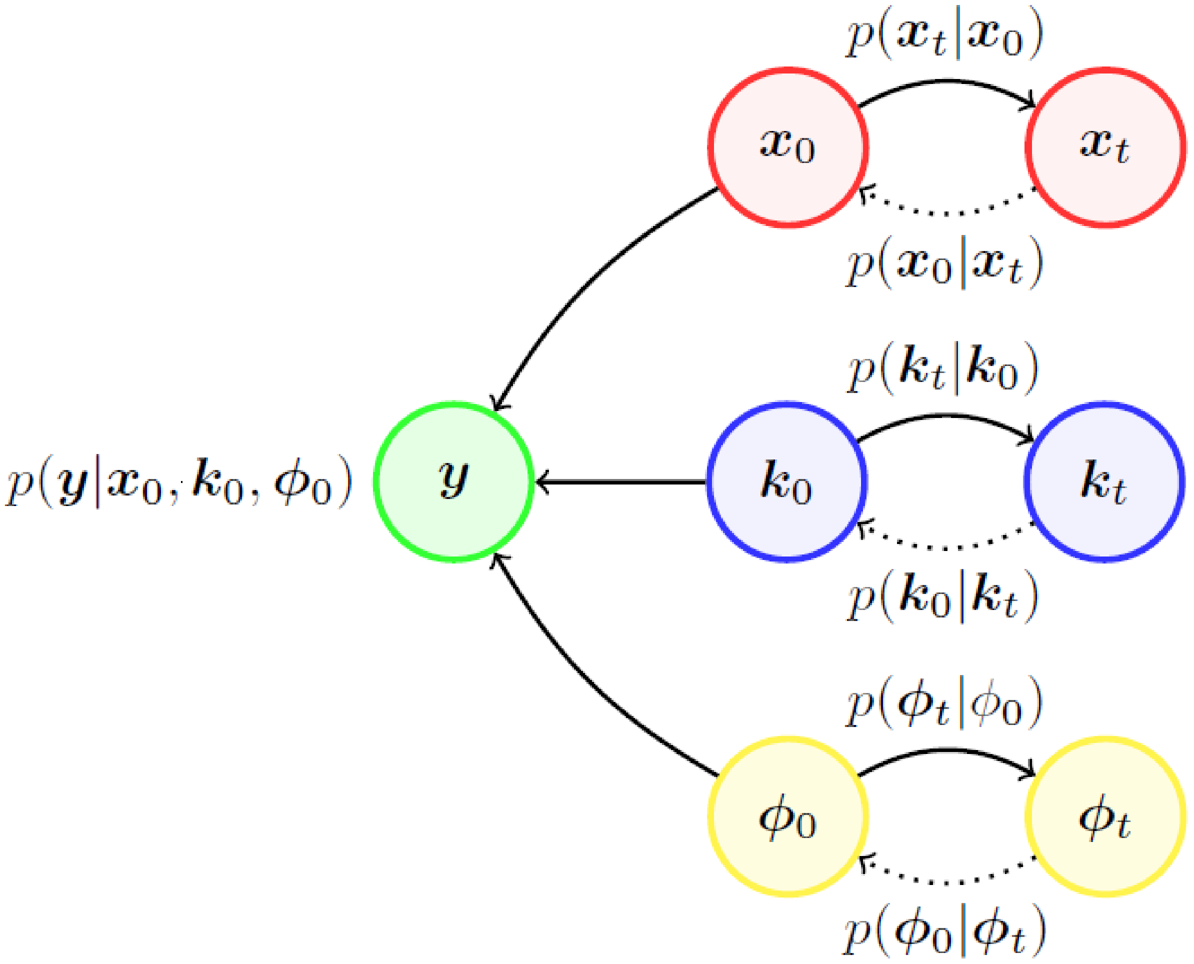
(b) Imaging through turbulence



General framework for Blind IP

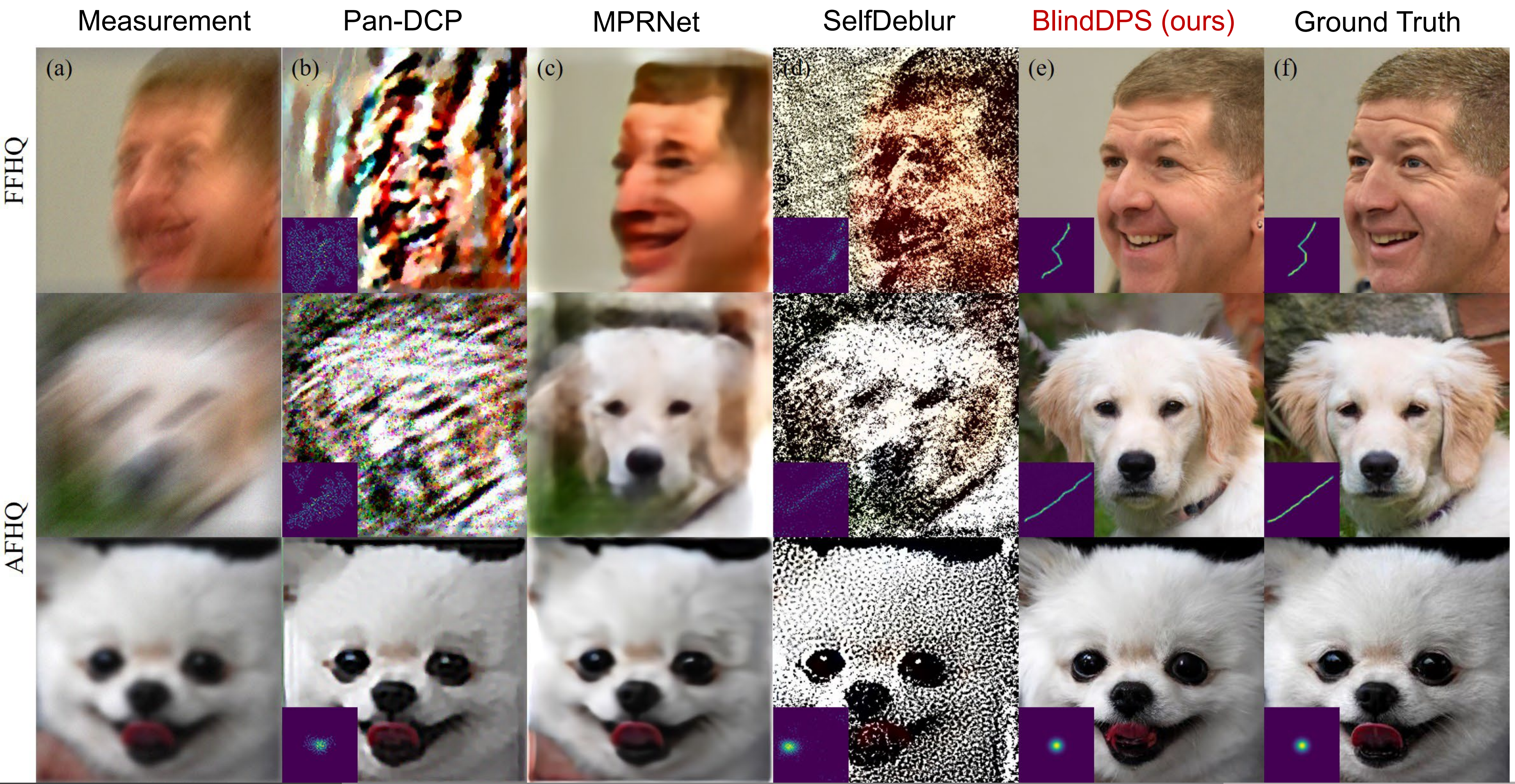


Blind deblurring

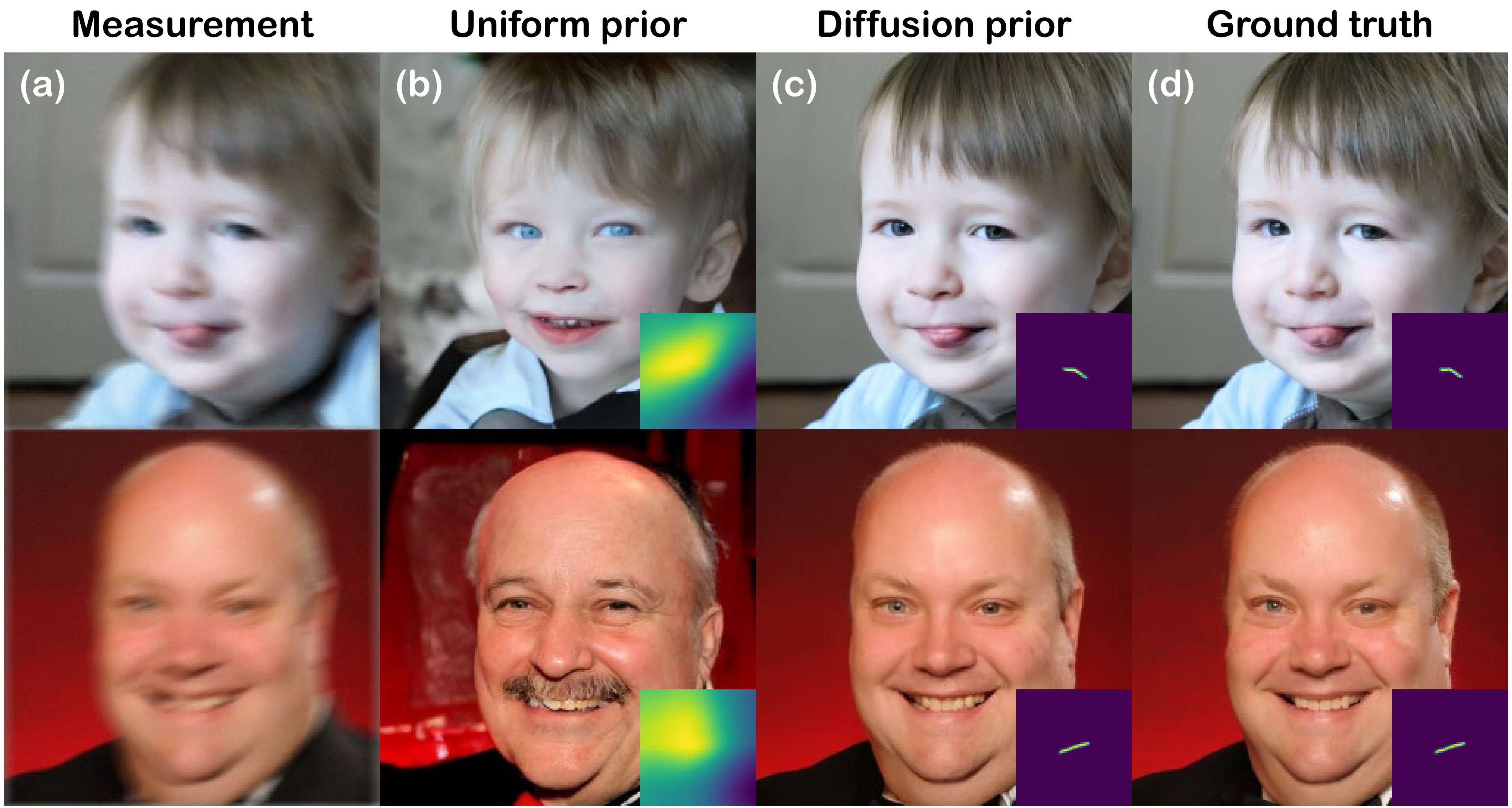


Imaging through turbulence

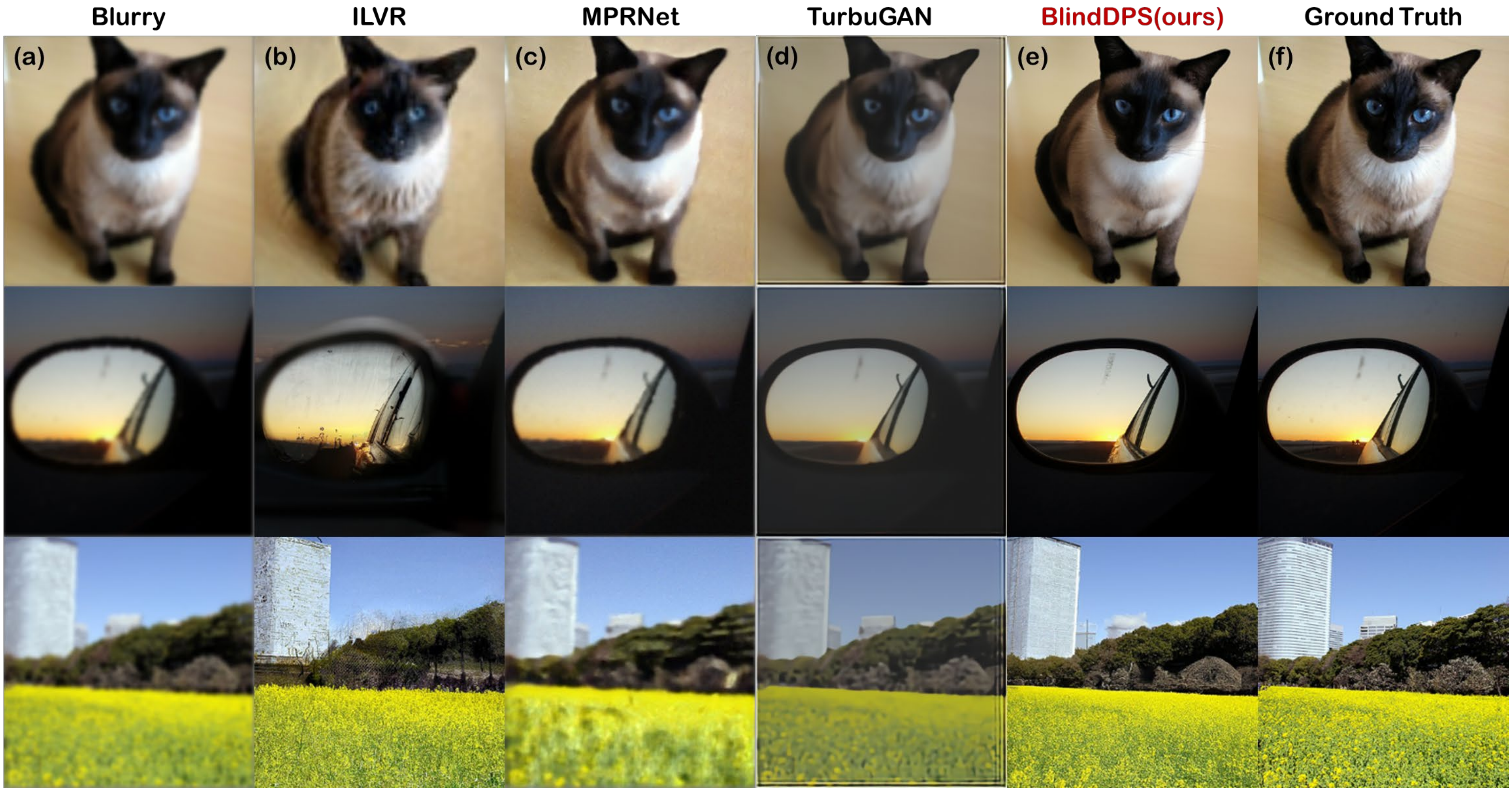
Results


Measurement
Pan-DCP
MPRNet
SelfDeblur
BlindDPS (ours)
Ground Truth
FFHQ
AFHQ
(a)
(b)
(c)
(d)
(e)
(f)

Ablation study against uninformative prior



Results



Blurry
ILVR
MPRNet
TurbuGAN
BlindDPS(ours)
Ground Truth
(a)
(b)
(c)
(d)
(e)
(f)

Thank you!

Project page: https://blinddps.github.io/blind-dps-project/

Code: https://github.com/BlindDPS/blind-dps

Paper: https://arxiv.org/abs/2211.10656