



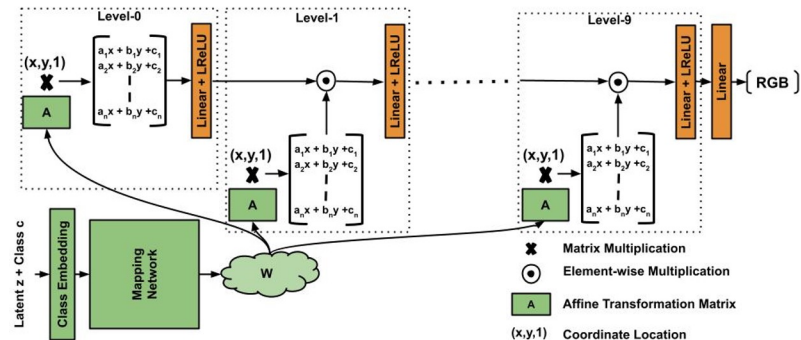
# Polynomial Implicit Neural Representations For Large Diverse Datasets

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# Overview

- **Implicit Neural Representations (INRs)** are effective for 2D or 3D scene representation and are further extended as a generative model.
- **Sinusoidal position encoding** : positional embedding can limit large dataset representation.
- We propose **Polynomial Implicit Neural Representation (Poly-INR)** and design an MLP model for the approximation of higher-order polynomials.
- Poly-INR as a generative model performs comparably to the state-of-the-art CNN-based generative models on the ImageNet dataset.



# Implicit Neural Representation (INR)

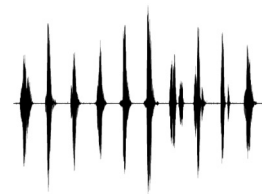
- **Classical signal representation:** discretize pixel value, point cloud, discretize amplitude.
- **Implicit neural representation (INR):** Multi-Layer Perceptron (MLP) is trained to generate the signal.
- **Benefits of such representation:**
  - Memory efficient
  - Better gradients or higher order derivatives computation
  - Solving inverse problem



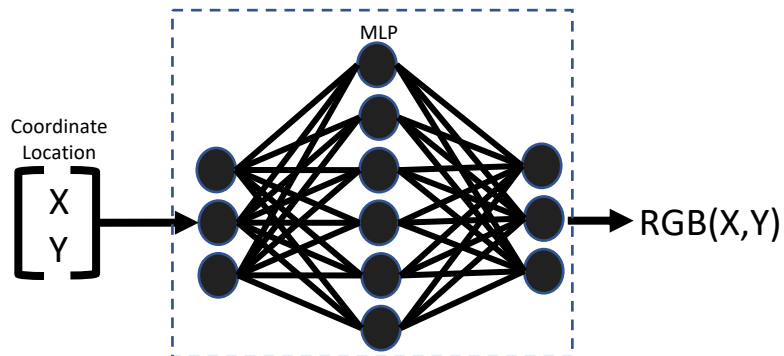
Image



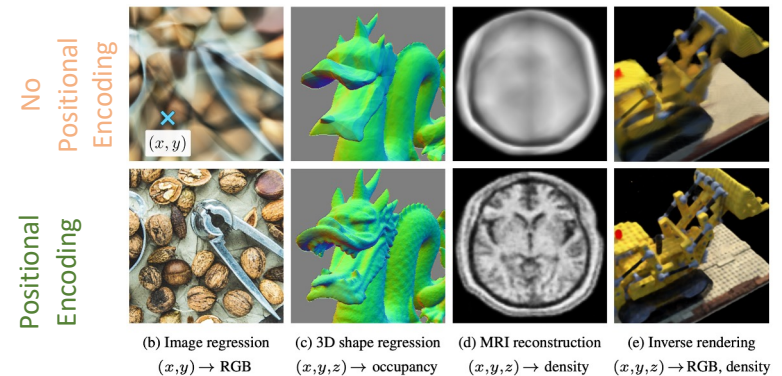
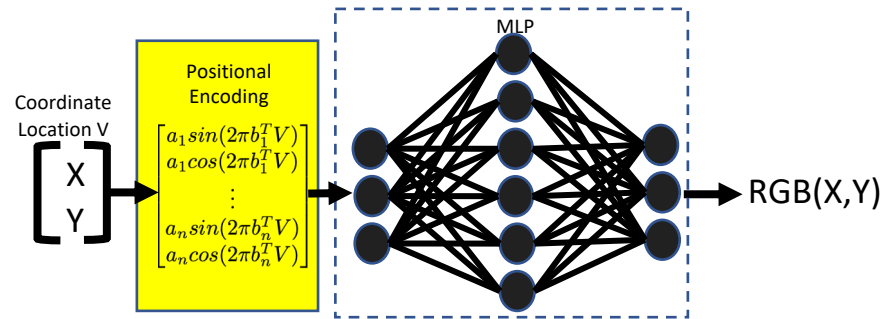
3D Shape



Audio



# Implicit Neural Representation (INR)



- ReLU-based MLP only retains low-frequency information.
- Periodic function based positional encoding is used for high frequency representation.
- **Positional encoding is limiting** for large dataset representation for two reasons:
  - Size of the encoding space is limited and low dimensional.
  - Conventional CNN based generative model first represents low frequency information like shape in the initial levels and progressively adds high frequency information.

# Polynomial Implicit Neural Representation

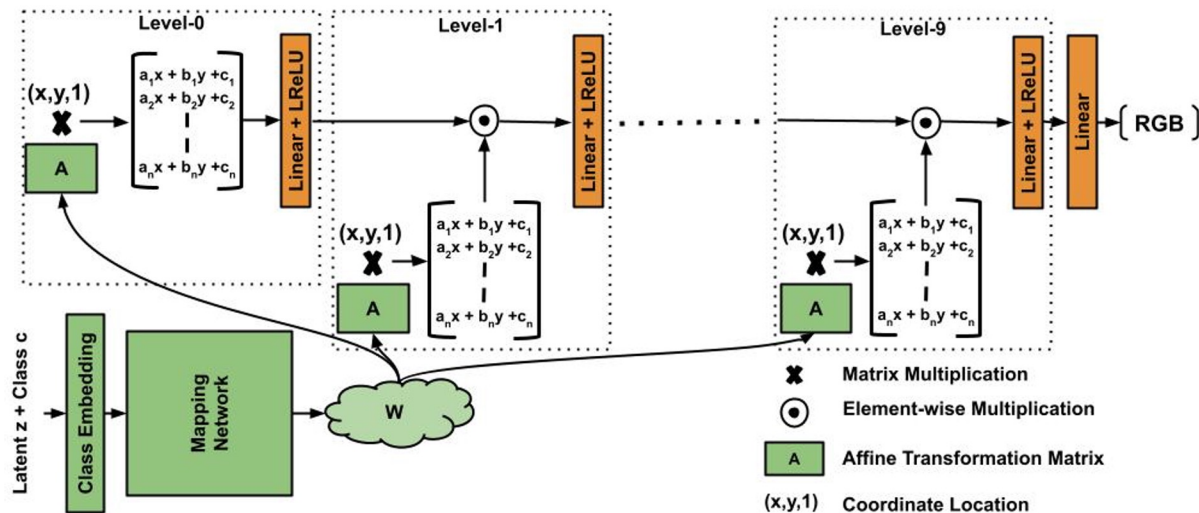
- Polynomial representation:

$$G(x, y) = g_{00} + g_{10}x + g_{01}y + \dots + g_{pq}x^p y^q$$

where  $(x, y)$  is the normalized pixel location and polynomial coefficients  $g_{pq}$  are parameterized by a latent vector  $z$ .

- Positional embedding of the form  $x^p y^q$  to approximate a higher-order polynomial can be limiting due to finite size of embedding space.
- Hence, we progressively increase the polynomial order in the network and let it learn the required orders.
- We use element-wise multiplication with the affine-transformed coordinate location at different levels, giving the network flexibility to increase the order as required.
- Model is trained as a Generative Adversarial Network (GAN).

# Polynomial Implicit Neural Representation



- **Mapping network** takes the latent code  $z \in R^{64}$  and maps it to the affine parameter space  $W \in R^{512}$ , consists of consists of an MLP with two linear layers.
- Synthesis network generates the RGB value for given pixel location.
- **Synthesis Network:**  $G_{syn} = \dots \sigma(W_2((A_2X) \odot \sigma(W_1((A_1X) \odot \sigma(W_0(A_0X))))))$ ,

# Quantitative Results

- Comparison against CNN-based GANs (BigGAN and StyleGAN-XL) and diffusion models (ADM and DiT-XL) on the ImageNet dataset.
- Metrics: Frechet Inception Distance (FID), Inception Score (IS), Precision (Pr) and Recall (Rec).

(a) ImageNet 128 × 128

Model	FID ↓	sFID ↓	rFID ↓	IS ↑	Pr ↑	Rec ↑
BigGAN	6.02	7.18	6.09	145.83	0.86	0.35
CDM	3.52	-	-	128.80	-	-
ADM	5.91	5.09	13.29	93.31	0.70	0.65
ADM-G	2.97	5.09	3.80	141.37	0.78	0.59
StyleGAN-XL	1.81	3.82	1.82	200.55	0.77	0.55
<b>Poly-INR</b>	<b>2.08</b>	<b>3.93</b>	<b>2.76</b>	<b>179.64</b>	<b>0.70</b>	<b>0.45</b>

(b) ImageNet 256 × 256

Model	FID ↓	sFID ↓	rFID ↓	IS ↑	Pr ↑	Rec ↑
BigGAN	6.95	7.36	75.24	202.65	0.87	0.28
ADM	10.94	6.02	125.78	100.98	0.69	0.63
ADM-G	3.94	6.14	11.86	215.84	0.83	0.53
DiT-XL/2-G	2.27	4.60	-	278.54	0.83	0.57
StyleGAN-XL	2.30	4.02	7.06	265.12	0.78	0.53
<b>Poly-INR</b>	<b>2.86</b>	<b>4.37</b>	<b>7.79</b>	<b>241.43</b>	<b>0.71</b>	<b>0.39</b>

(c) ImageNet 512 × 512

Model	FID ↓	sFID ↓	rFID ↓	IS ↑	Pr ↑	Rec ↑
BigGAN	8.43	8.13	312.00	177.90	0.88	0.29
ADM	23.24	10.19	561.32	58.06	0.73	0.60
ADM-G	3.85	5.86	210.83	221.72	0.84	0.53
DiT-XL/2-G	3.04	5.04	-	240.82	0.84	0.54
StyleGAN-XL	2.41	4.06	51.54	267.75	0.77	0.52
<b>Poly-INR</b>	<b>3.81</b>	<b>5.06</b>	<b>54.31</b>	<b>267.44</b>	<b>0.70</b>	<b>0.34</b>

(d) Number of parameters in millions (M)

Model	64 <sup>2</sup>	128 <sup>2</sup>	256 <sup>2</sup>	512 <sup>2</sup>
BigGAN	-	141.0	164.3	164.7
ADM	296.0	422.0	554.0	559.0
DiT-XL	-	-	675.0	675.0
StyleGAN-XL	134.4	158.7	166.3	168.4
<b>Poly-INR</b>	<b>46.0</b>	<b>46.0</b>	<b>46.0</b>	<b>46.0</b>

Sauer, Axel, Katja Schwarz, and Andreas Geiger. "Stylegan-xl: Scaling stylegan to large diverse datasets." ACM SIGGRAPH 2022 conference proceedings. 2022.

Dhariwal, Prafulla, and Alexander Nichol. "Diffusion models beat gans on image synthesis." Advances in Neural Information Processing Systems 34 (2021)

Peebles, William, and Saining Xie. "Scalable Diffusion Models with Transformers." 2022

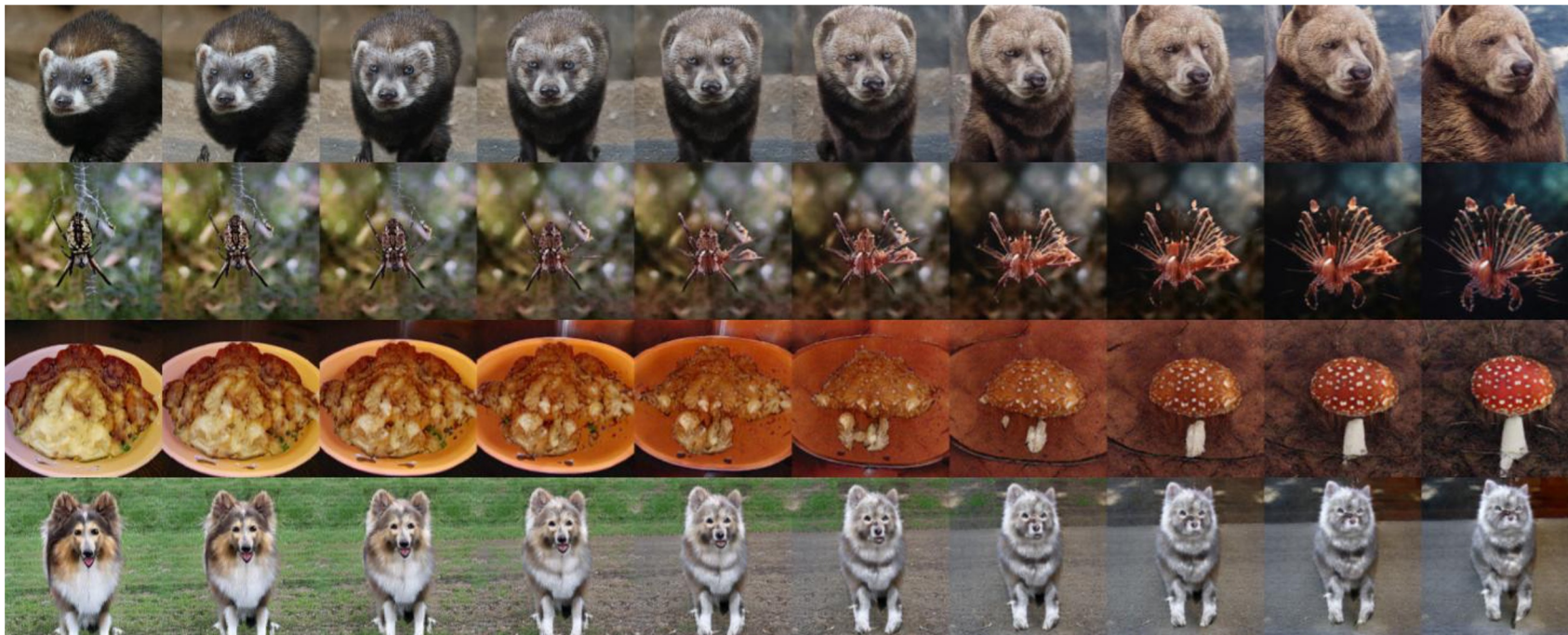
# Generated Samples





# Interpolation

Poly-INR provides smooth interpolation in affine parameters space.



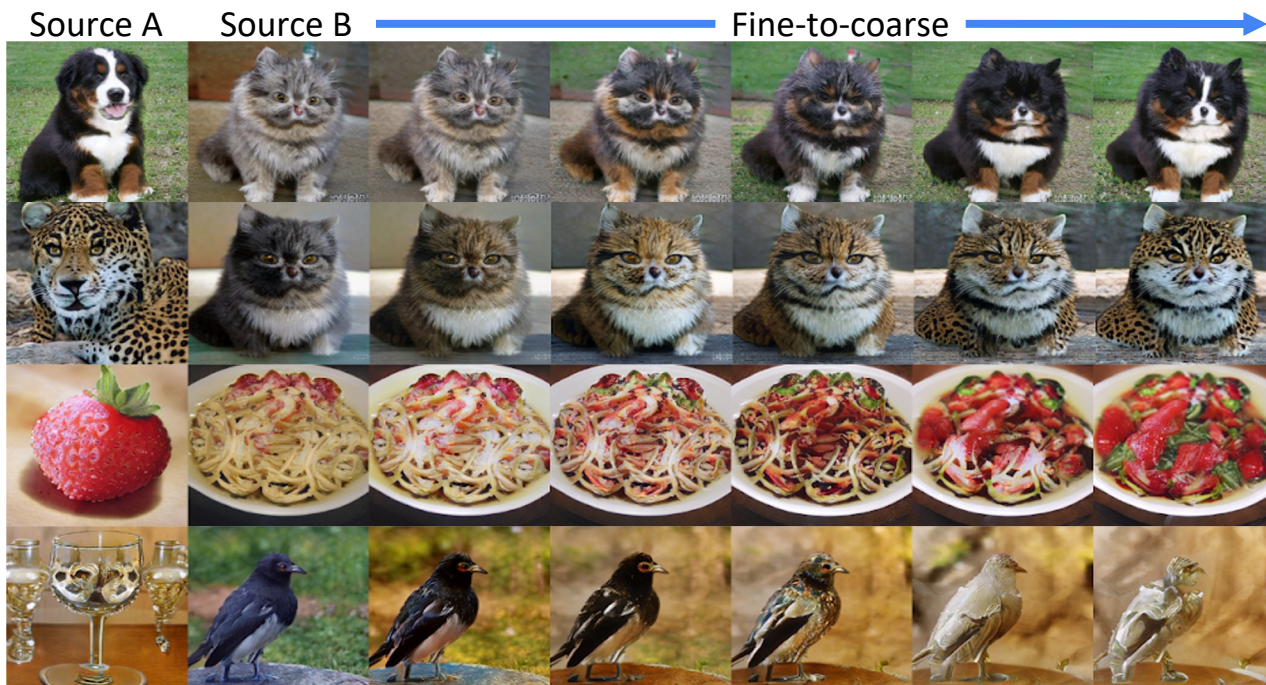
# Shape Mixing

Copying the affine parameters of source A to source B at lower levels (0-5) brings change in the shape



# Style Mixing

Copying the affine parameters of source A to source B at higher levels (5 - 9) brings a change in the style



**Thank You**