

Marching-Primitives: Shape Abstraction from Signed Distance Function

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4.1 MB



2.6 MB



6.2 KB



8.1 MB



1.2 MB



3.3 KB

High-resolution Mesh

Marching Cubes

Marching Primitives

Unlike previous works which extract polygonal meshes from an SDF, we present the first method, named Marching-Primitives, to obtain a primitive-based abstraction directly from an SDF

Visualization of SDF

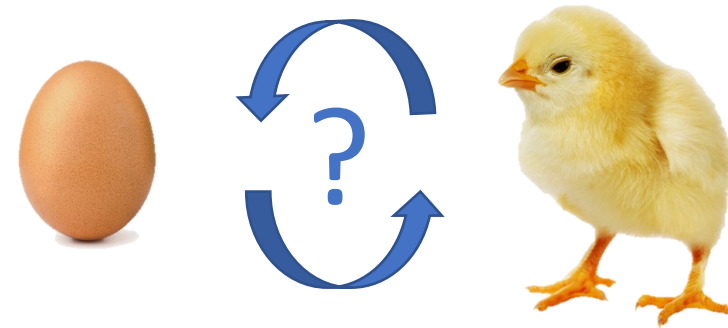


— Target Shape

— Primitives

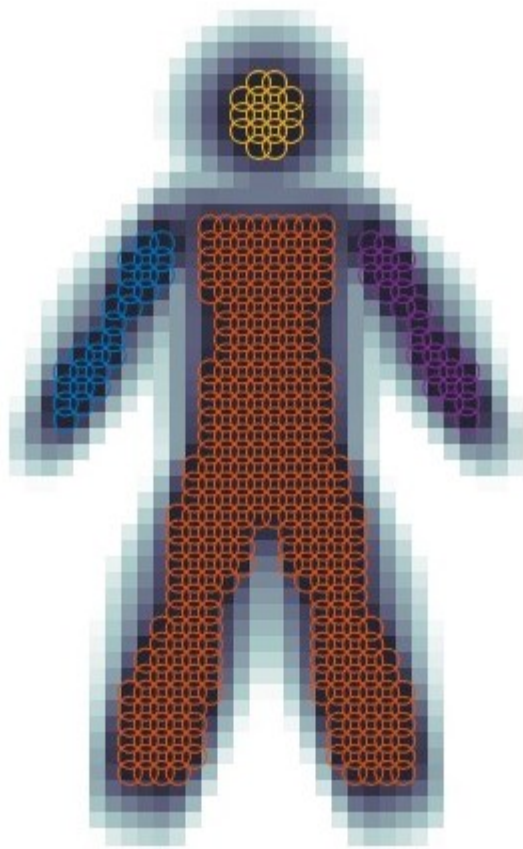
Combination of Geometric Primitives

- Voxels where geometric primitives can be extracted?



- What geometry best fit the underlying voxels?

Connectivity-Marching



— Target Shape

— Primitives

Thresholds $T^c \doteq \{t_1^c, t_2^c, \dots\}, \quad t_1^c = \min_{\mathbf{x}_i \in \mathbf{V}} d(\mathbf{x}_i), \quad t_{m+1}^c = \alpha t_m^c$

Isosurfaces $S_m = \{\mathcal{S}_k, k = 1, 2, \dots, |S_m|\}$

Volumes of Interest $\bar{S}_m = \{\mathcal{S}_k \in S_m, |\mathcal{S}_k| \geq N_c\} \subseteq S_m$

Probabilistic Primitive-Marching

Generative Model

$$p(d_i | \boldsymbol{\theta}_k, z_{ik}) = \left(\frac{\mathbb{1}_{d_i \in [-t, 0)}}{t} \right)^{1-z_{ik}} \mathcal{N}(d_i | d_{\boldsymbol{\theta}_k}(\mathbf{x}_i), \sigma^2)^{z_{ik}}$$

Bayesian Correspondence

$$p(z_{ik} | \boldsymbol{\theta}_k, d_i) = \frac{p(d_i | \boldsymbol{\theta}_k, z_{ik}) p(z_{ik})}{\sum_{z_{ik} \in \{0,1\}} p(d_i | \boldsymbol{\theta}_k, z_{ik}) p(z_{ik})}$$

Primitive Update

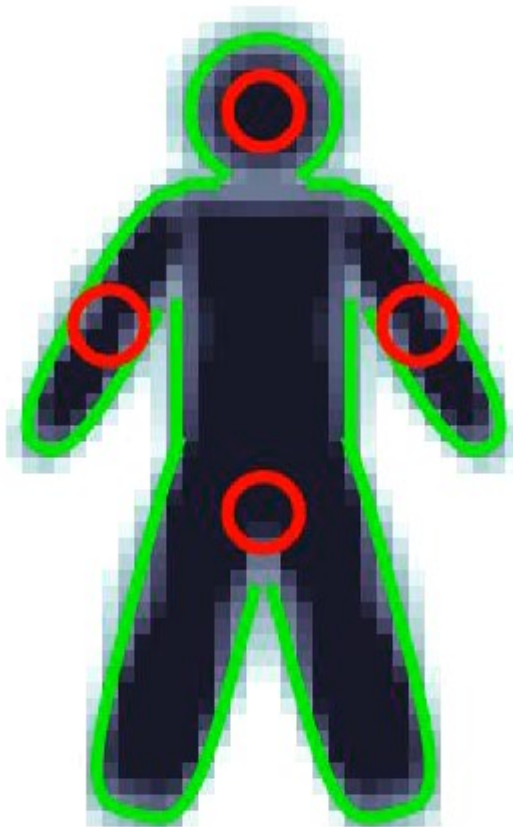
$$\boldsymbol{\theta}_k = \arg \min_{\boldsymbol{\theta}_k} \sum_{\mathbf{x}_i \in \mathbf{V}_a} P_{ik} \|d_{\boldsymbol{\theta}_k}(\mathbf{x}_i) - d_i\|_2^2$$

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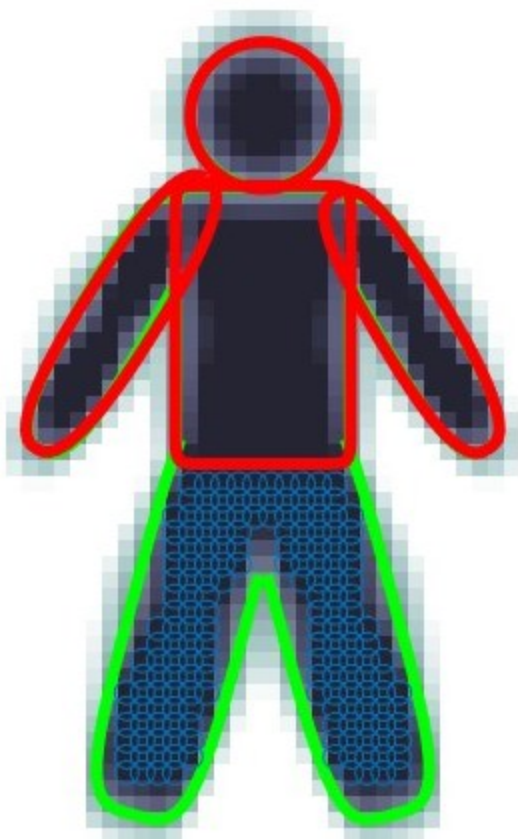
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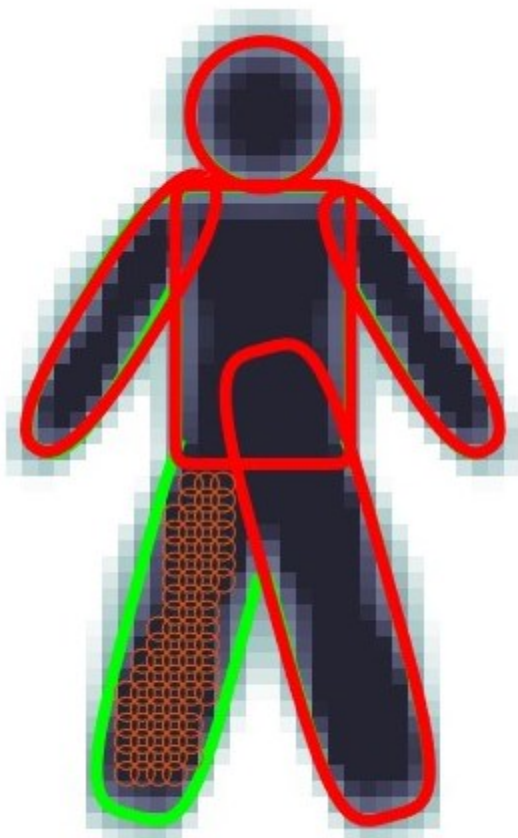
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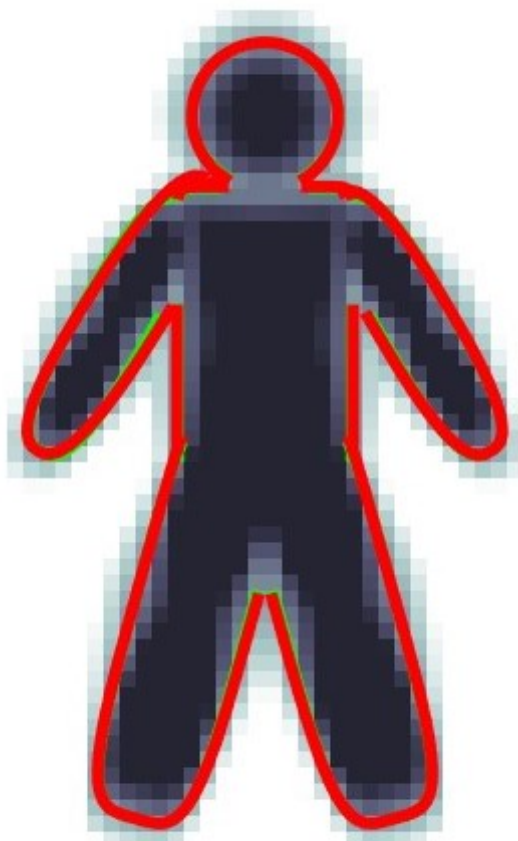
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Everything needed to describe the chair is here (973 bytes)



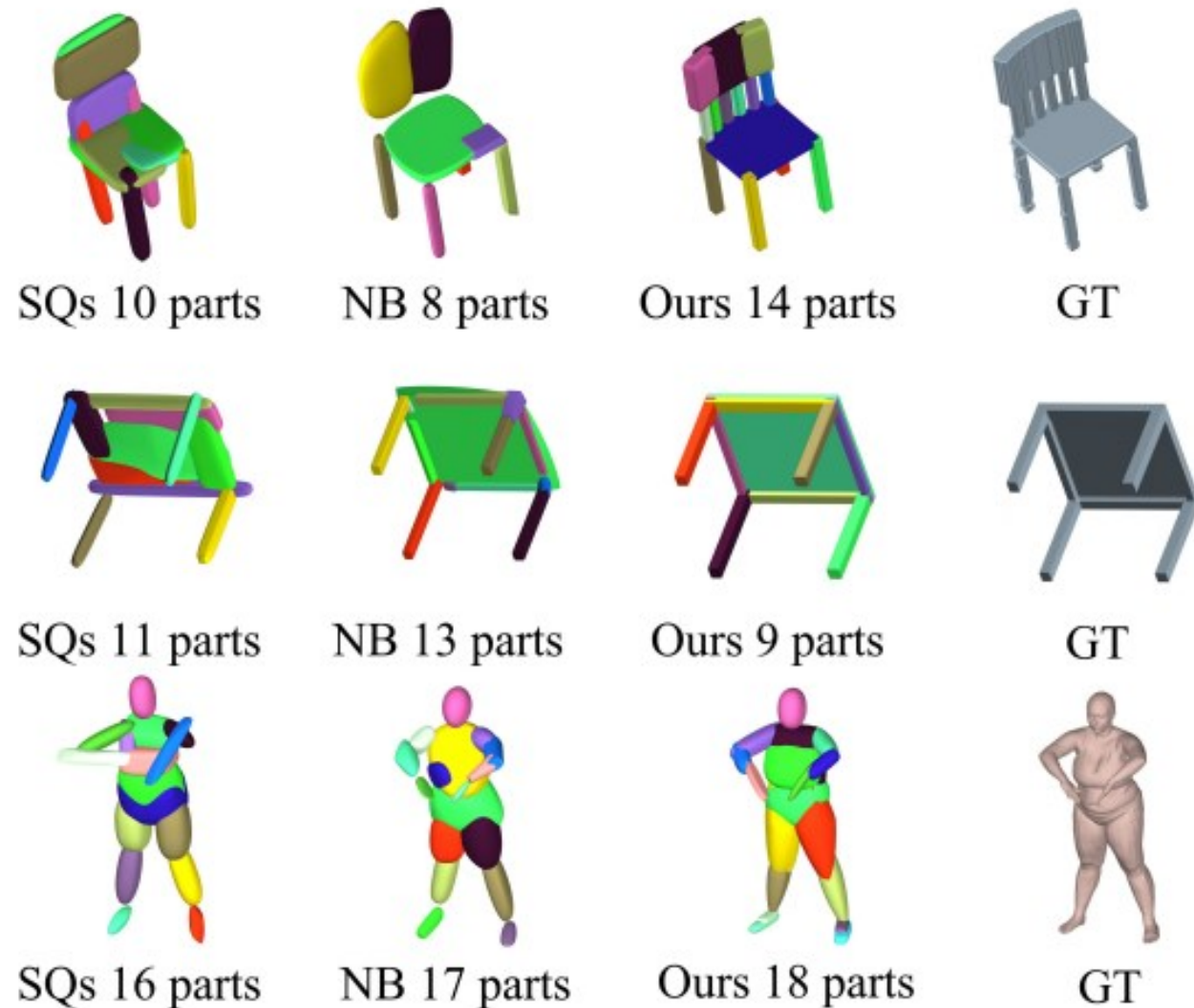
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0.0029	0.0474	0.4049	0.0960	0.0496	-4.4022e-09	-1.2613e-06	0.0028	0.0101	-0.0493	-0.4243
0.0451	0.9389	0.0621	0.0461	0.4283	1.5708	-0.4694	1.5708	0.0101	0.4366	0.3647
0.1380	0.4053	0.0424	0.0540	0.4421	8.2131e-04	2.6413e-04	1.0749e-04	-0.3770	-0.3078	-0.0535
0.0439	1.1220	0.0605	0.0447	0.4284	1.5708	-0.7662	1.5708	0.0101	0.5648	0.4054
0.1330	0.3491	0.0621	0.0762	0.3753	1.6425	-1.5333	3.0725	-0.3523	-0.4419	-0.4263
0.3340	0.2095	0.4446	0.0438	0.0516	-1.9746	-1.5703	0.4072	0.3987	-0.3053	-0.0523
0.0207	0.5037	0.0565	0.0407	0.4284	1.5708	-0.4782	1.5708	0.0101	0.3124	0.3273
0.3719	0.0822	0.0533	0.4021	0.0611	8.1752e-04	-0.0018	-0.2432	-0.3680	-0.4267	0.3561
0.1312	0.3489	0.0762	0.0621	0.3749	-0.0027	0.0026	-1.6082	0.3725	-0.4421	-0.4262
6.3959e-04	0.4127	0.0576	0.0884	0.4297	0.0109	-1.5818e-04	-7.4037e-04	-0.3723	-0.0591	-0.0759
0.0546	0.1936	0.0502	0.0964	0.4295	1.5708	-1.5654	1.5708	0.0101	-0.0498	0.3033
0.0944	0.2263	0.0708	0.0415	0.4285	1.5708	-0.3324	1.5708	0.0101	0.7660	0.4734
0.3714	0.0817	0.0533	0.4020	0.0611	-8.2808e-04	0.0018	-0.2432	0.3882	-0.4267	0.3561
9.1380e-04	0.4206	0.0582	0.0883	0.4297	3.1325	-7.3484e-05	7.6954e-04	0.3931	-0.0591	-0.0759
0.0512	0.2392	0.4285	0.0638	0.0494	1.5707	-0.2980	1.5670	-0.3641	0.4173	0.3847
0.1423	0.3905	0.0648	0.0548	0.3694	-1.5462	-1.2718	-0.0225	0.3891	0.3797	0.3729
0.1958	0.4215	0.0558	0.0642	0.4251	-3.1416	0.0026	1.2721	0.3898	0.4193	0.3856

Category	Chamfer- L_1				IoU			
	SQs [31]	NB [47]	MPE(Ours)	MPS(Ours)	SQs [31]	NB [47]	MPE(Ours)	MPS(Ours)
airplane	0.037	0.023	0.021	0.019	0.441	0.671	0.731	0.768
bench	0.056	0.028	0.020	0.020	0.238	0.579	0.730	0.819
bottle	0.047	0.033	0.026	0.017	0.686	0.665	0.886	0.924
cabinet	0.059	0.036	0.037	0.028	0.394	0.666	0.840	0.948
can	0.066	0.036	0.036	0.022	0.706	0.553	0.908	0.950
chair	0.068	0.027	0.023	0.020	0.300	0.685	0.785	0.871
lamp	0.072	0.029	0.022	0.021	0.234	0.589	0.750	0.802
speaker	0.064	0.041	0.037	0.033	0.346	0.656	0.858	0.920
mailbox	0.095	0.026	0.026	0.024	0.333	0.694	0.802	0.905
rifle	0.038	0.020	0.019	0.019	0.446	0.732	0.744	0.811
sofa	0.054	0.037	0.029	0.023	0.497	0.726	0.857	0.940
table	0.070	0.024	0.024	0.022	0.247	0.745	0.818	0.932
phone	0.040	0.021	0.023	0.021	0.681	0.872	0.891	0.947
watercraft	0.048	0.032	0.022	0.022	0.465	0.618	0.793	0.836
mean	0.057	0.028	0.024	0.022	0.368	0.674	0.793	0.870

Table 1. Quantitative results on Shapenet. MPE and MPS are short for Marching-Primitives with ellipsoids and superquadrics, respectively

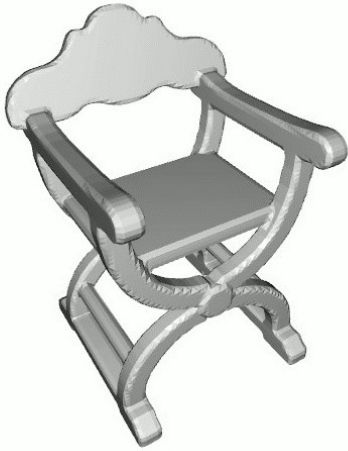


	SQs	NB	MPE (Ours)	MPS (Ours)
Chamfer- L_1	0.0435	0.0303	0.0301	0.0291
IoU	0.7417	0.8107	0.8568	0.8598



More Examples

Marching Cubes



1.64MB

Marching Primitives (Ours)



1.67KB

Marching Cubes



1.11MB

Marching Primitives (Ours)



1.27KB

Marching Cubes



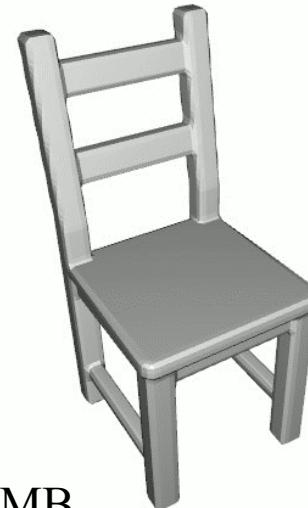
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Marching Primitives (Ours)



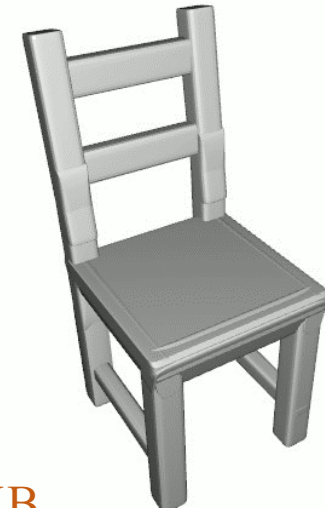
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Marching Cubes



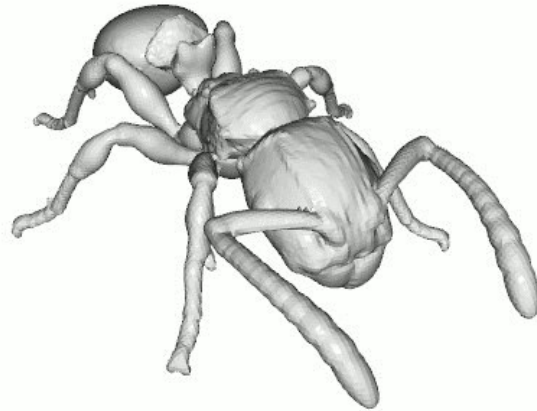
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Marching Primitives (Ours)

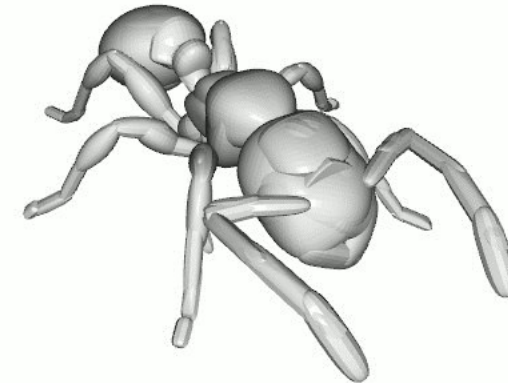


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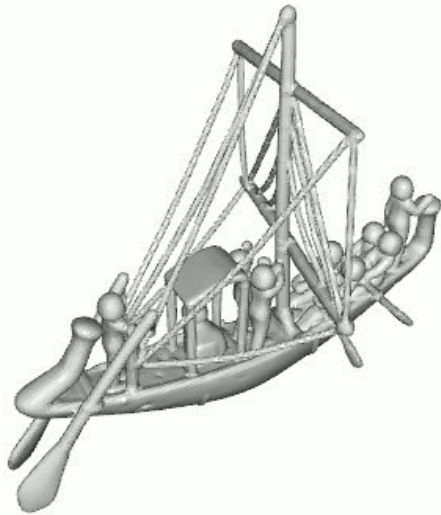
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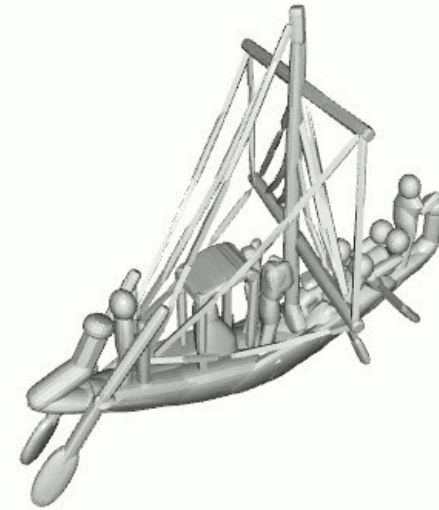
Marching Primitives (Ours)



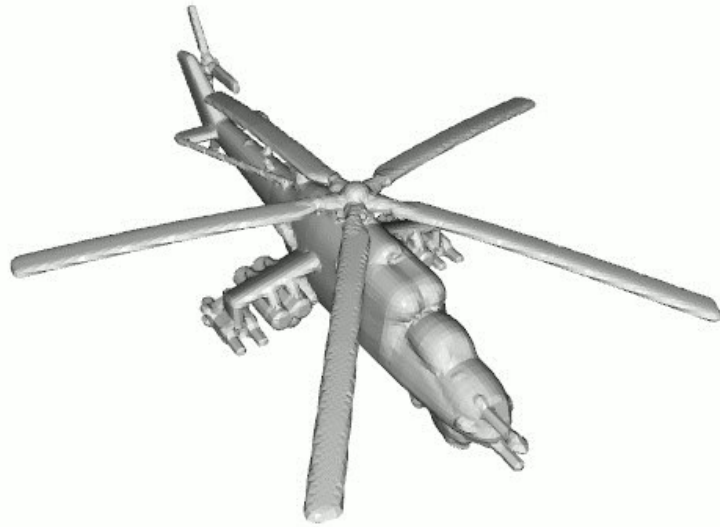
Marching Cubes



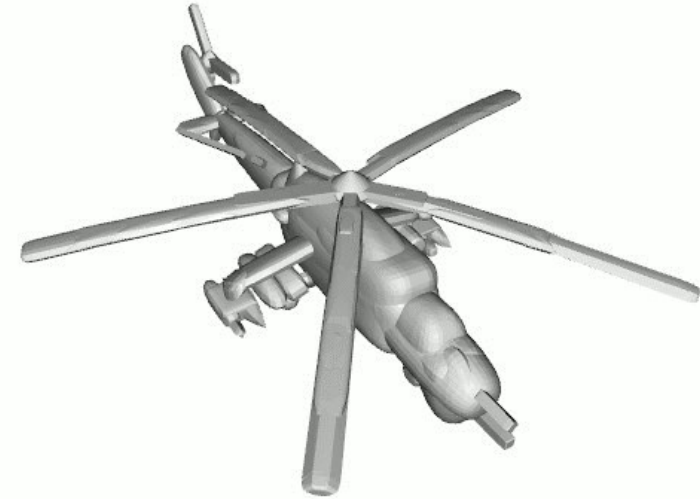
Marching Primitives (Ours)



Marching Cubes



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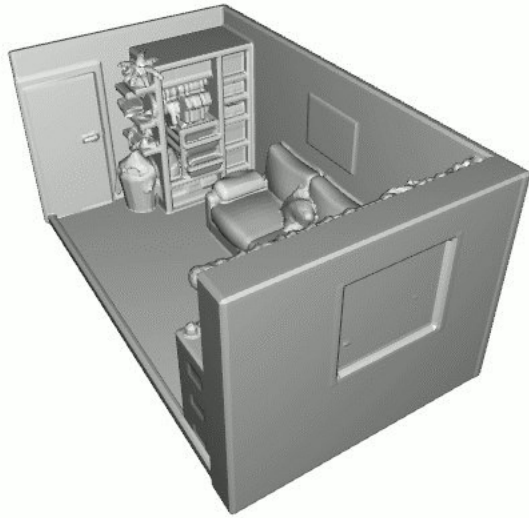
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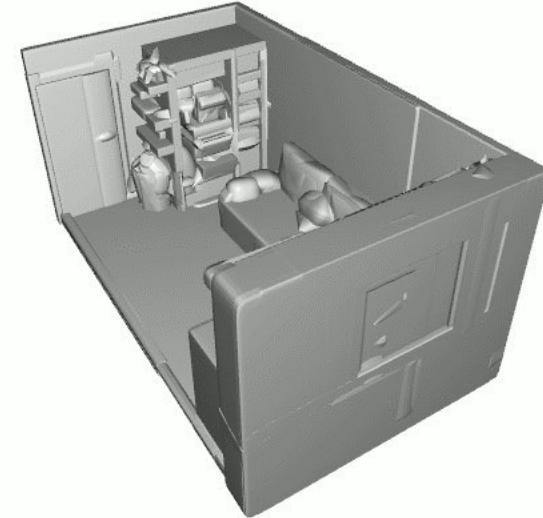
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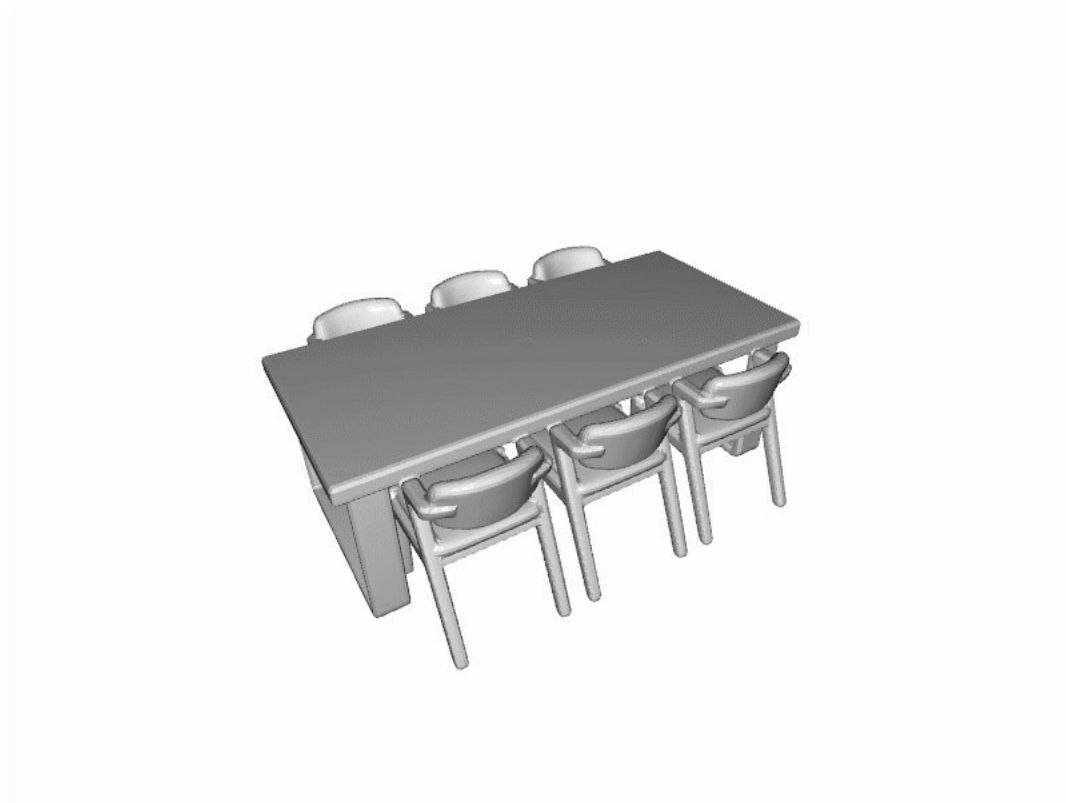
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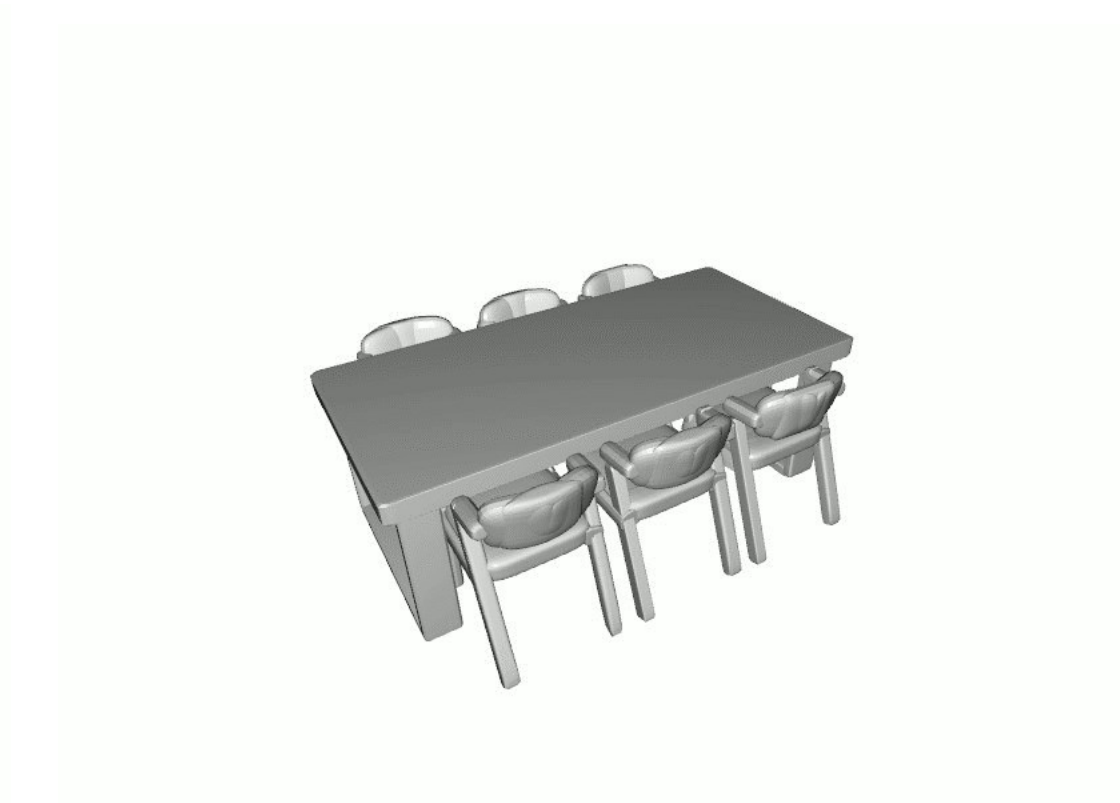
Marching Primitives (Ours)



Marching Cubes



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Our code is open-sourced at
<https://github.com/ChirikjianLab/Marching-Primitives>
or scan the QR code

