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INSTITUTE OF TECHNOLOGY

# Learning the Distribution of Errors in Stereo Matching for Joint Disparity and Uncertainty Estimation

*Liyan Chen, Weihan Wang, Philippas Mordohai*

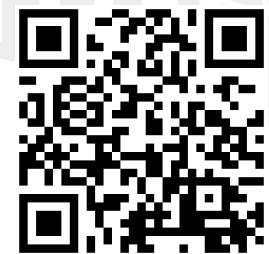
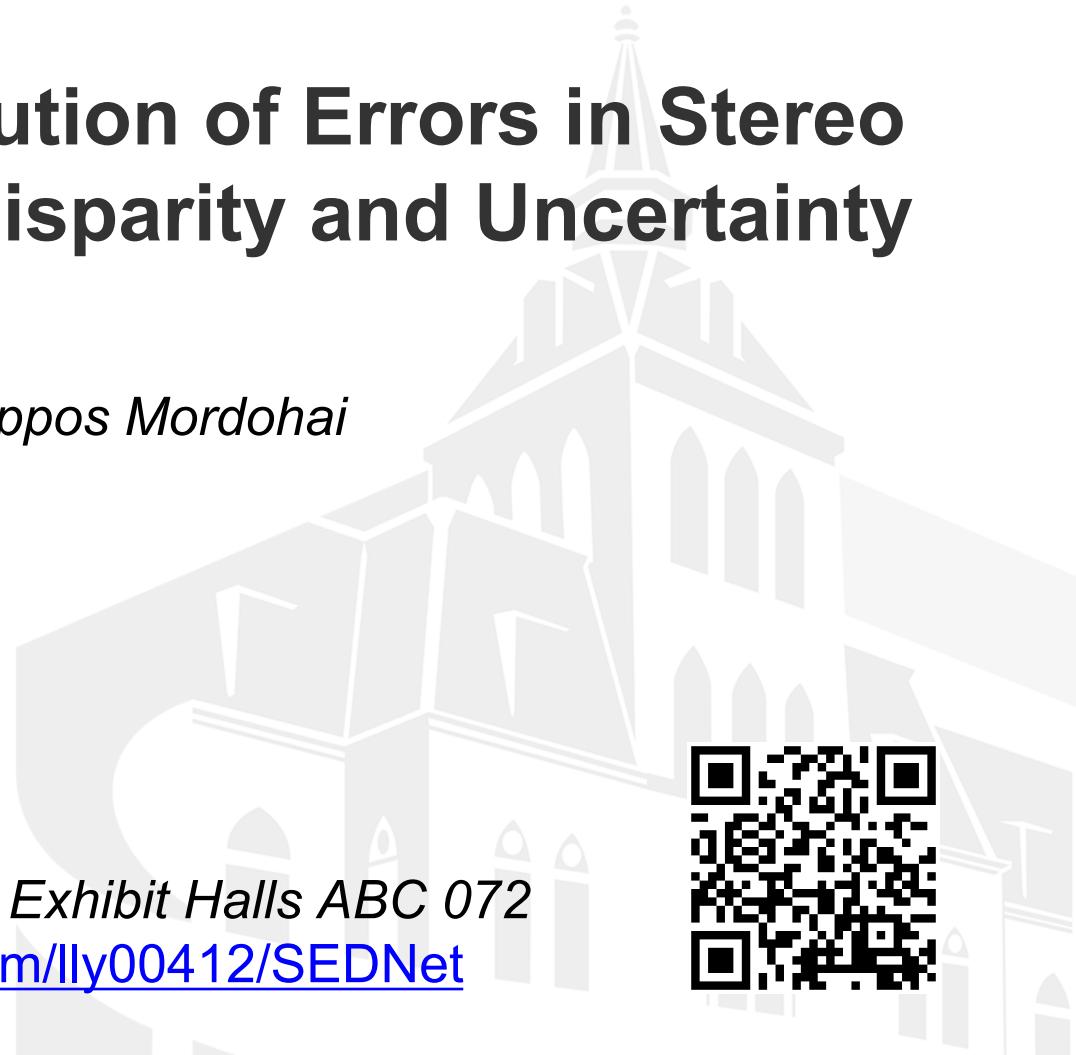


Email: [Ichen39@stevens.edu](mailto:Ichen39@stevens.edu)

Poster Section : THU-AM-072

Poster Location : West Building Exhibit Halls ABC 072

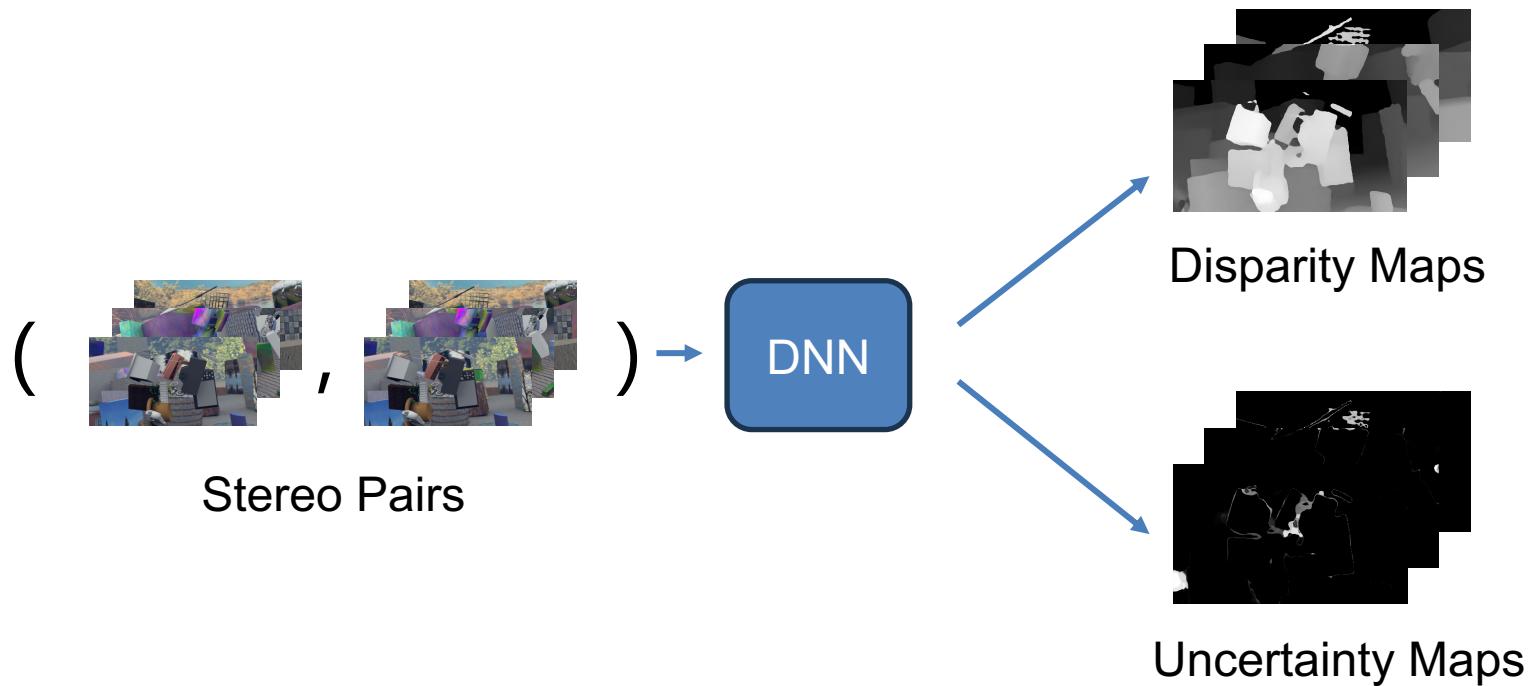
Implementation: <https://github.com/lly00412/SEDNet>



# Contributions

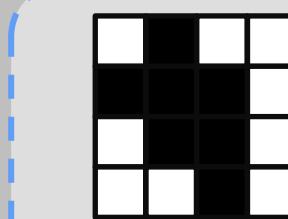
- ❖ A novel joint estimation network, **SEDNet (*Stereo Error Distribution Network*)** predicts **disparity** as well as the **aleatoric uncertainty**.
- ❖ A **differentiable soft-histogramming** technique used to **approximate the distributions** of disparity errors and estimated uncertainties.
- ❖ A **matching error loss** based on KL divergence applied on histograms obtained with the above technique to **improve the precision of uncertainty estimation**.

# Introduction

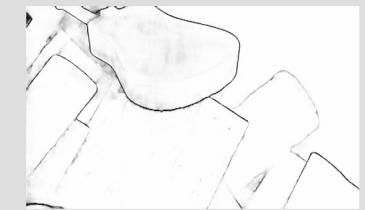


**Joint estimation** of disparity and uncertainty / confidence benefits both tasks due to **multi-task learning**.

## Confidence Estimation



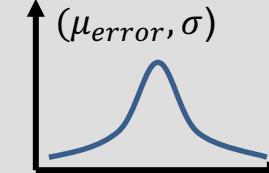
binary variable



*conf\_map*

w/ BCE loss

## Uncertainty Estimation

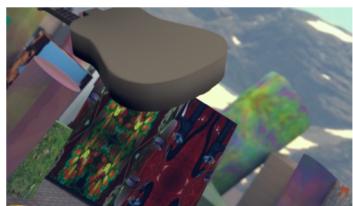


continuous variable



*uncert\_map*

w/ NLL or KL loss

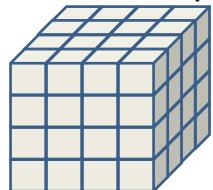


*rgb\_img*

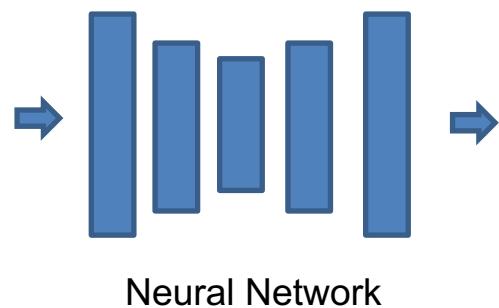


*disp\_map*  
+

Additional input



*cost\_volume*

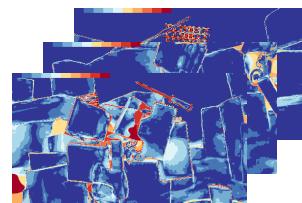


Neural Network

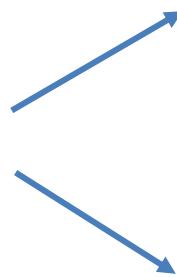
# Problem



Disparity Maps



Error Maps



Predict the magnitude  
of per-pixel error?



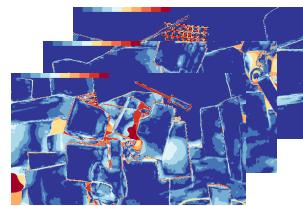
Predict the uncertainty  
per pixel



# Objective



Disparity Maps



Error Maps

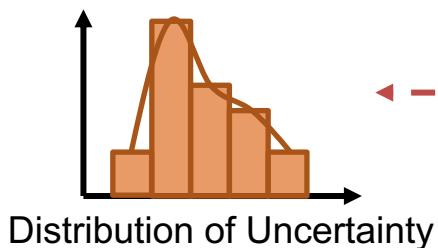
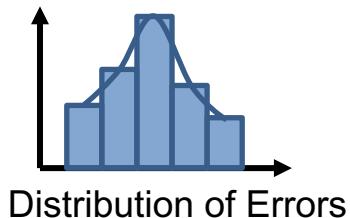
Predict the magnitude  
of per pixel error?



Predict the uncertainty  
per pixel



To train an **uncertainty** estimator whose outputs **follow the same distribution** as the **true errors of the disparity** estimator.



(Kendall and Gal, 2017)



Uncertainty Maps

# Aleatoric Uncertainty Estimation

(Kendall and Gal, 2017) & (Ilg et al., 2018)

Minimizing **NLL loss** per pixel

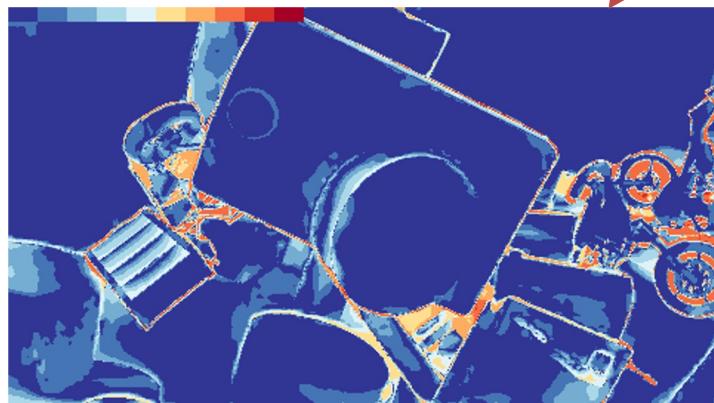
$$\mathcal{L}_{log} = \frac{1}{n} \sum_{i=1}^n \frac{|\hat{d}^{(i)} - d^{(i)}|}{\exp(s^{(i)})} + \frac{1}{n} \sum_{i=1}^n s^{(i)}$$

# Aleatoric Uncertainty Estimation

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Error Map

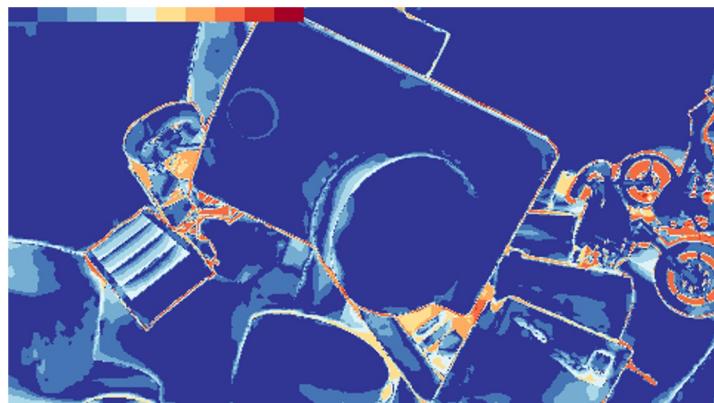
# Aleatoric Uncertainty Estimation

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i.e.  $\sigma^{(i)}$

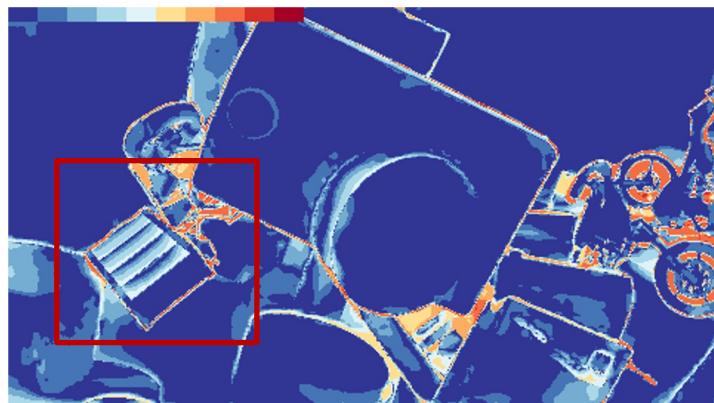
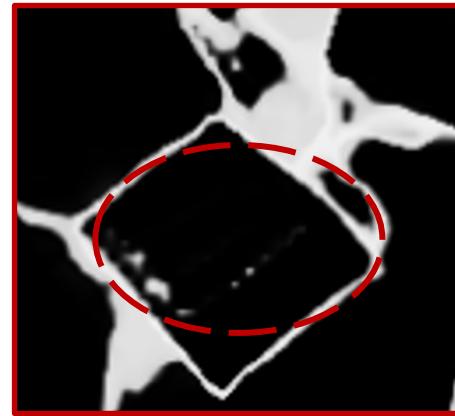
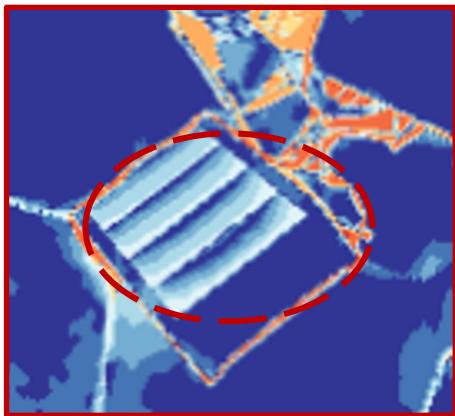


Error Map



Uncertainty Map

# Mismatch Between Error and Uncertainty

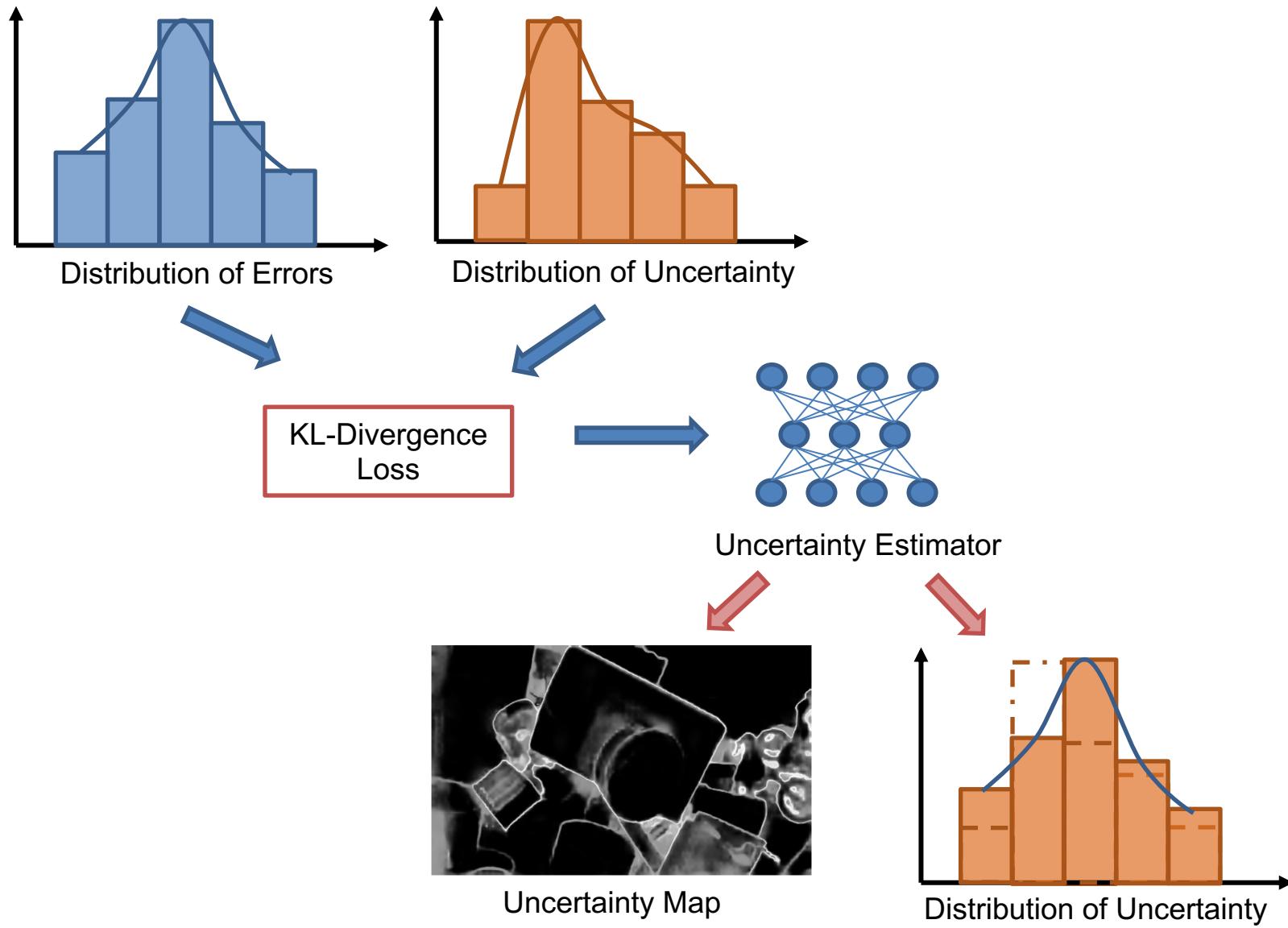


Error Map

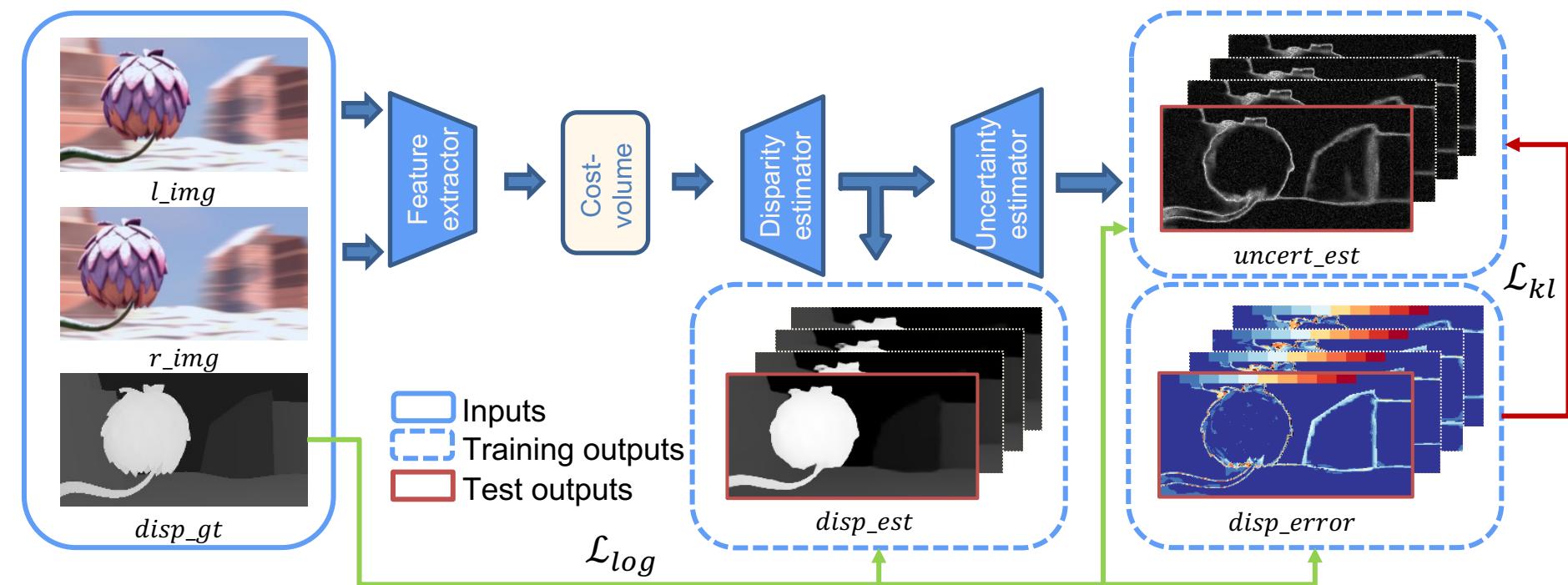


Uncertainty Map

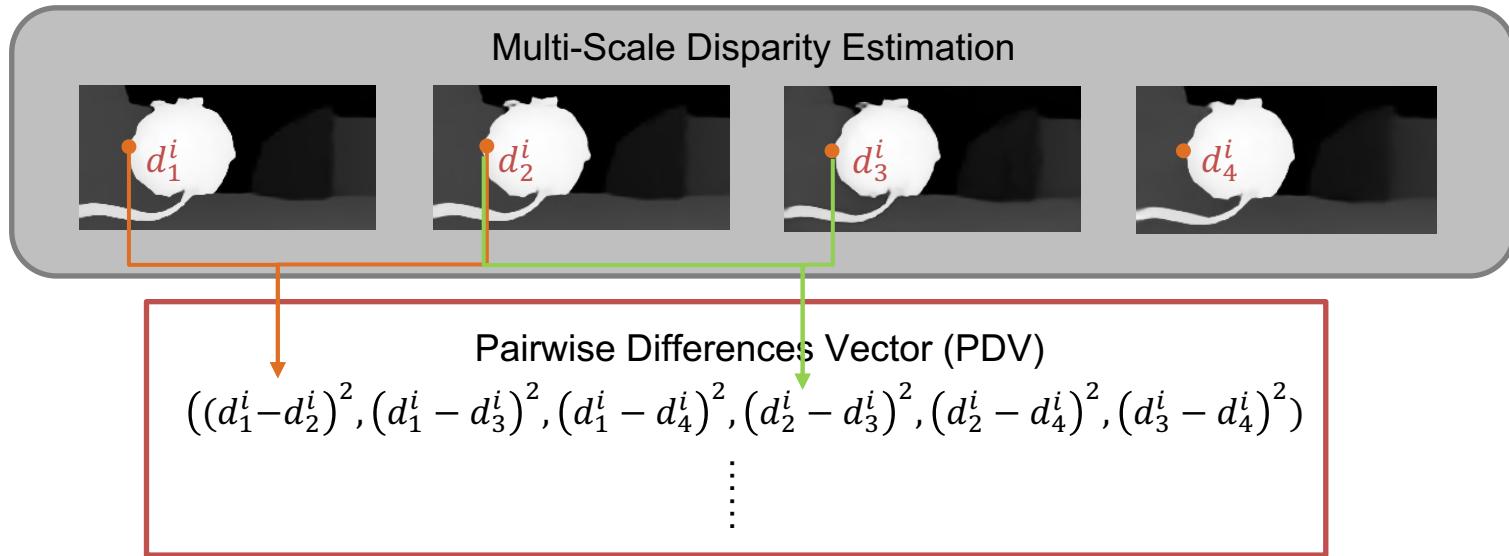
# Learning the Distribution of Errors



# SEDNet - Pipeline Overview

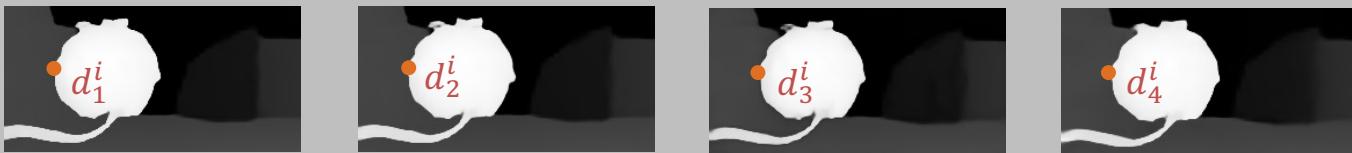


# SEDNet – Uncertainty Estimator

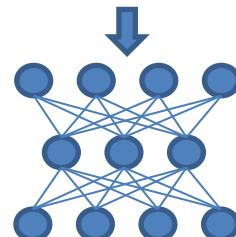


**Squared differences** between  
disparity estimates at different scales

# SEDNet – Uncertainty Estimator



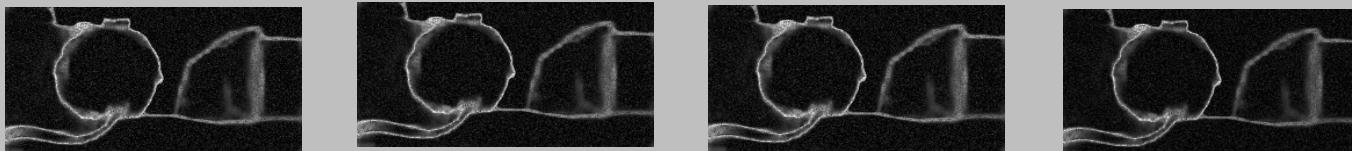
Pairwise Differences Vector (PDV)



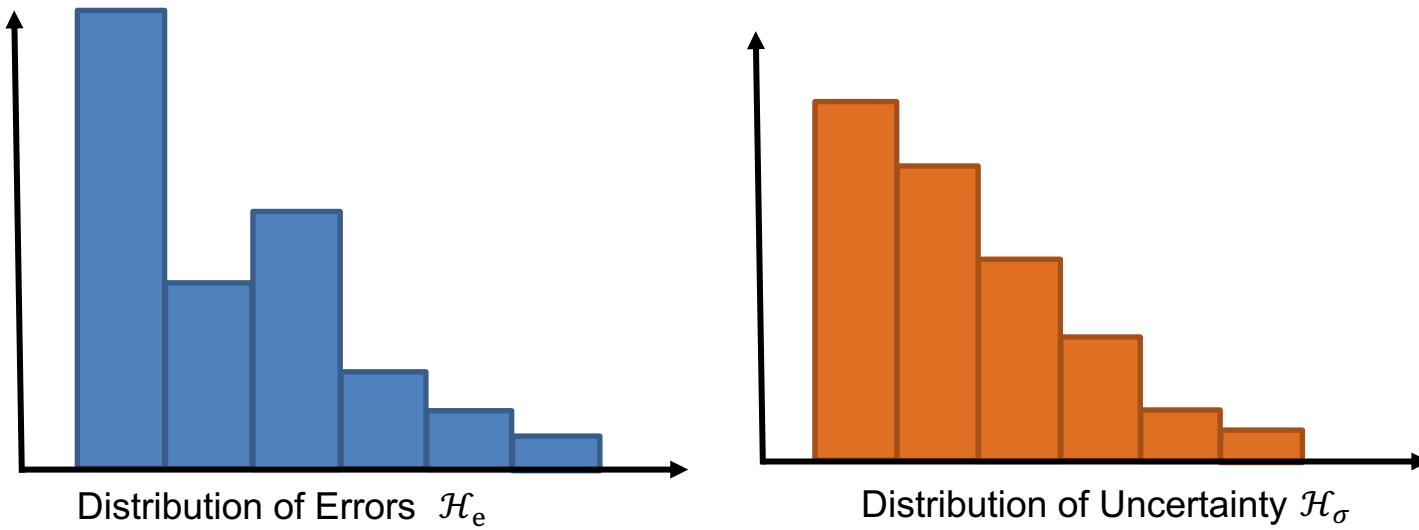
3-Layer MLP



Multi-Scale Uncertainty Estimation

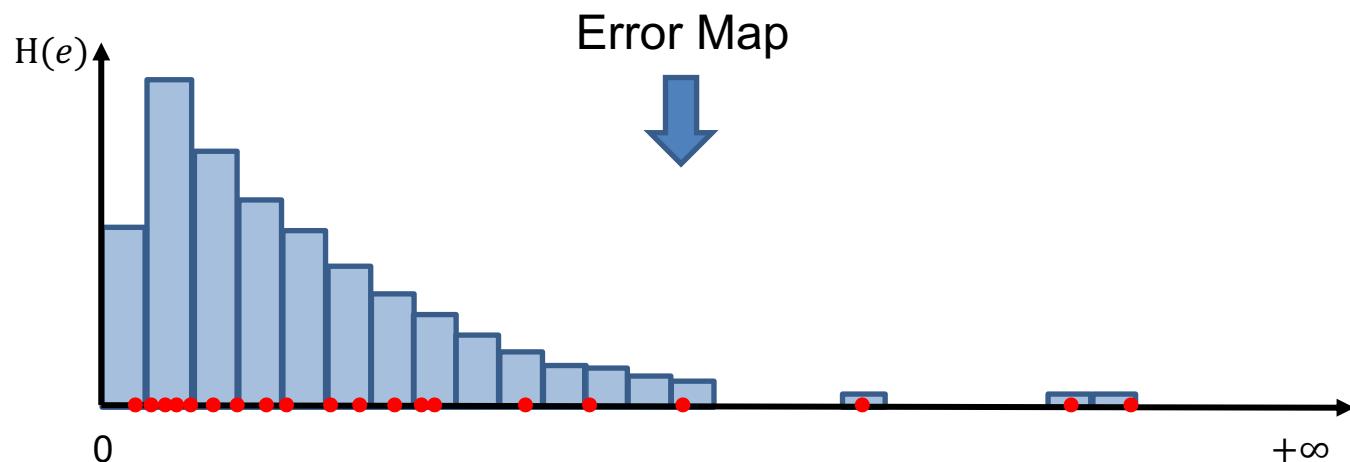
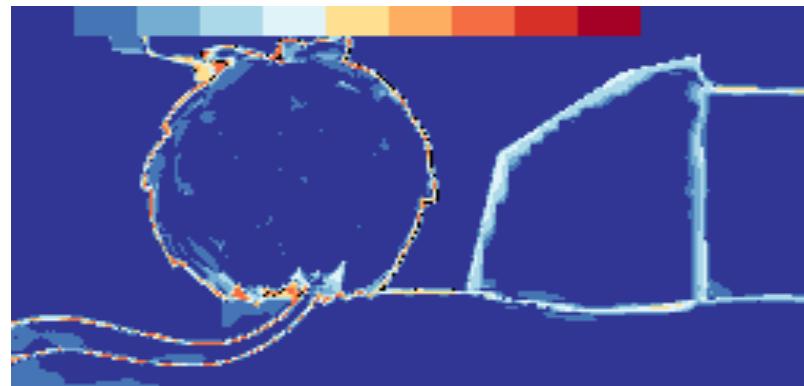


# Matching the Distribution of Error

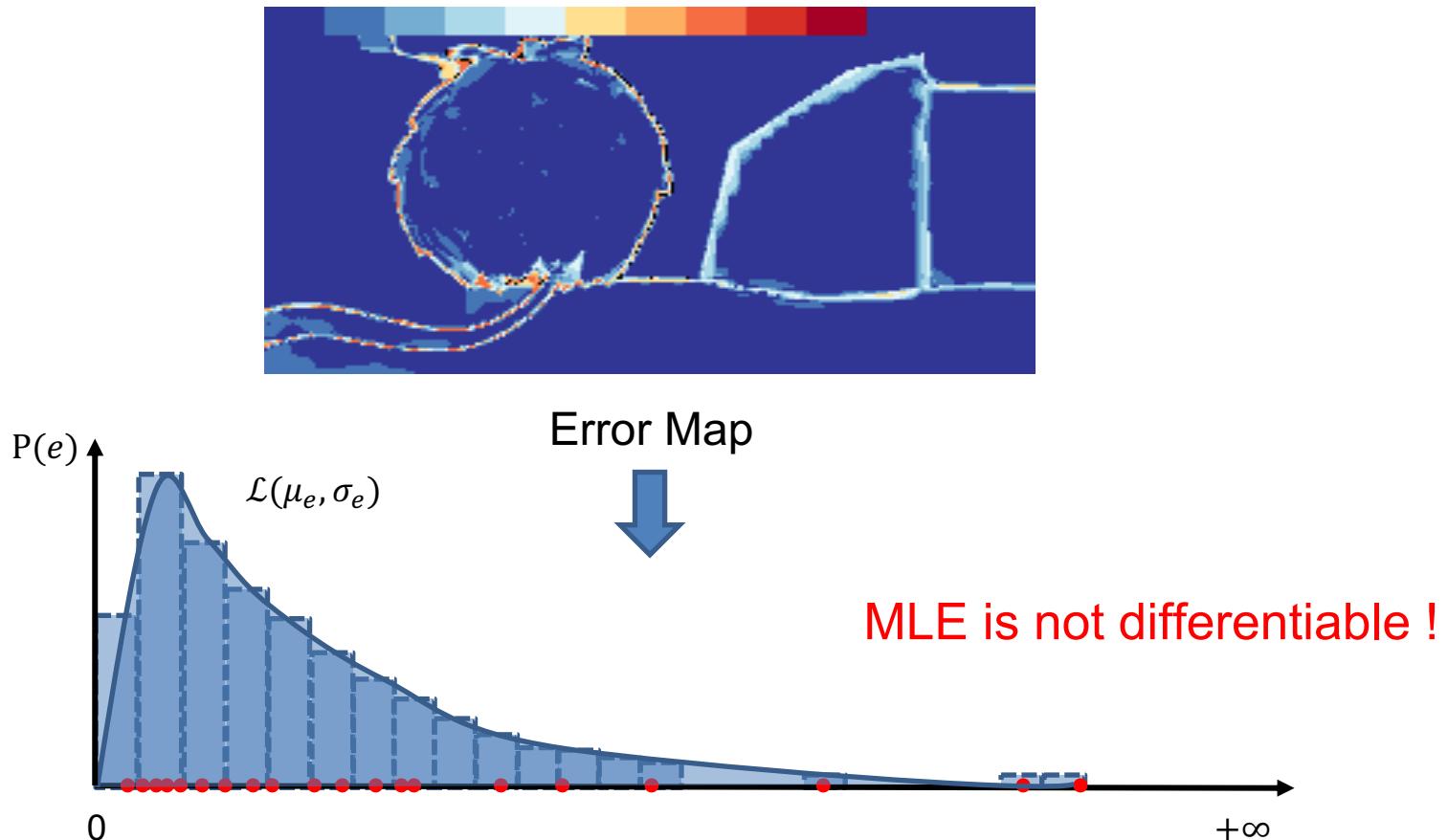


To prepare **the inputs to the matching error loss**, we need to build the distributions.

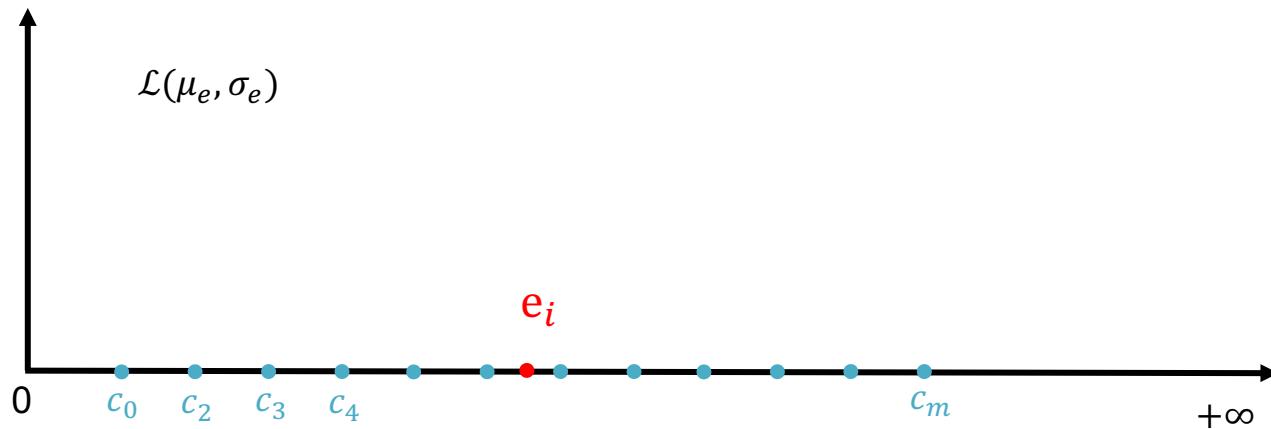
# Convert Error Map to Distribution



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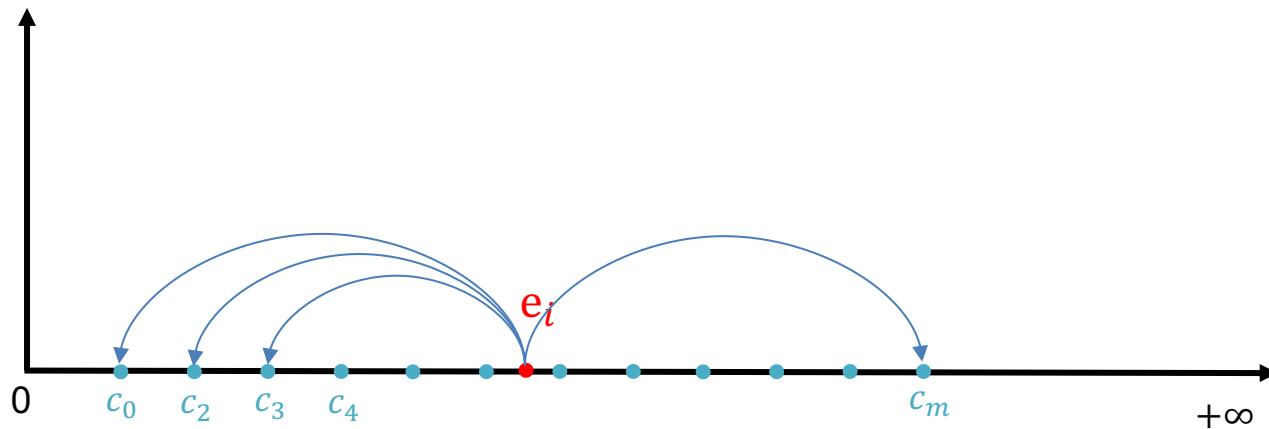


# Soft Histogramming



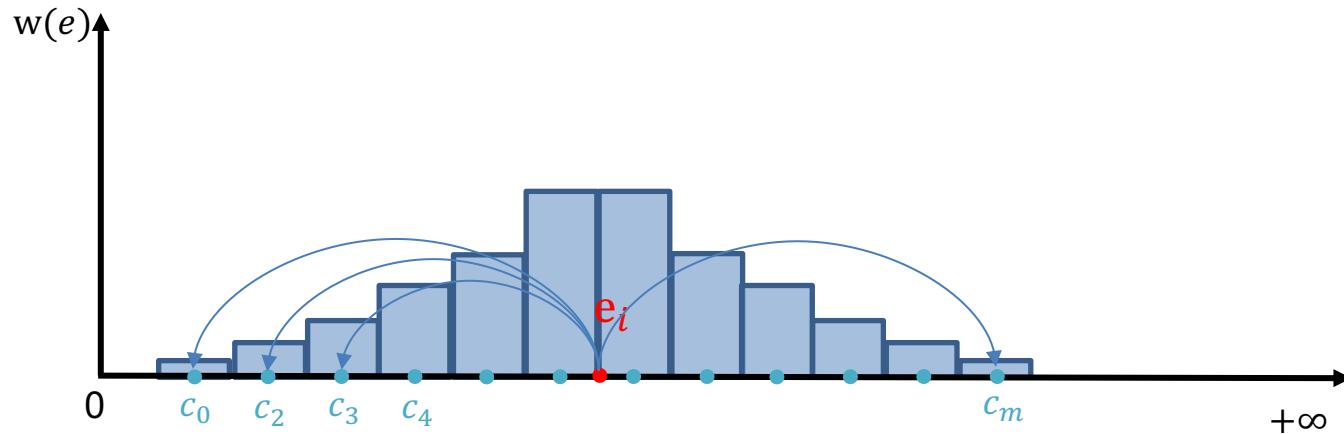
1. Pick  $m$  bin centers, where  $c_1 = \mu_e$ ,  $c_m = \mu_e + m\sigma_e$

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2. Compute  $L2$  distances between  $e_i$  and bin centers:  
 $d_{i,1}, d_{i,2}, \dots, d_{i,m}$

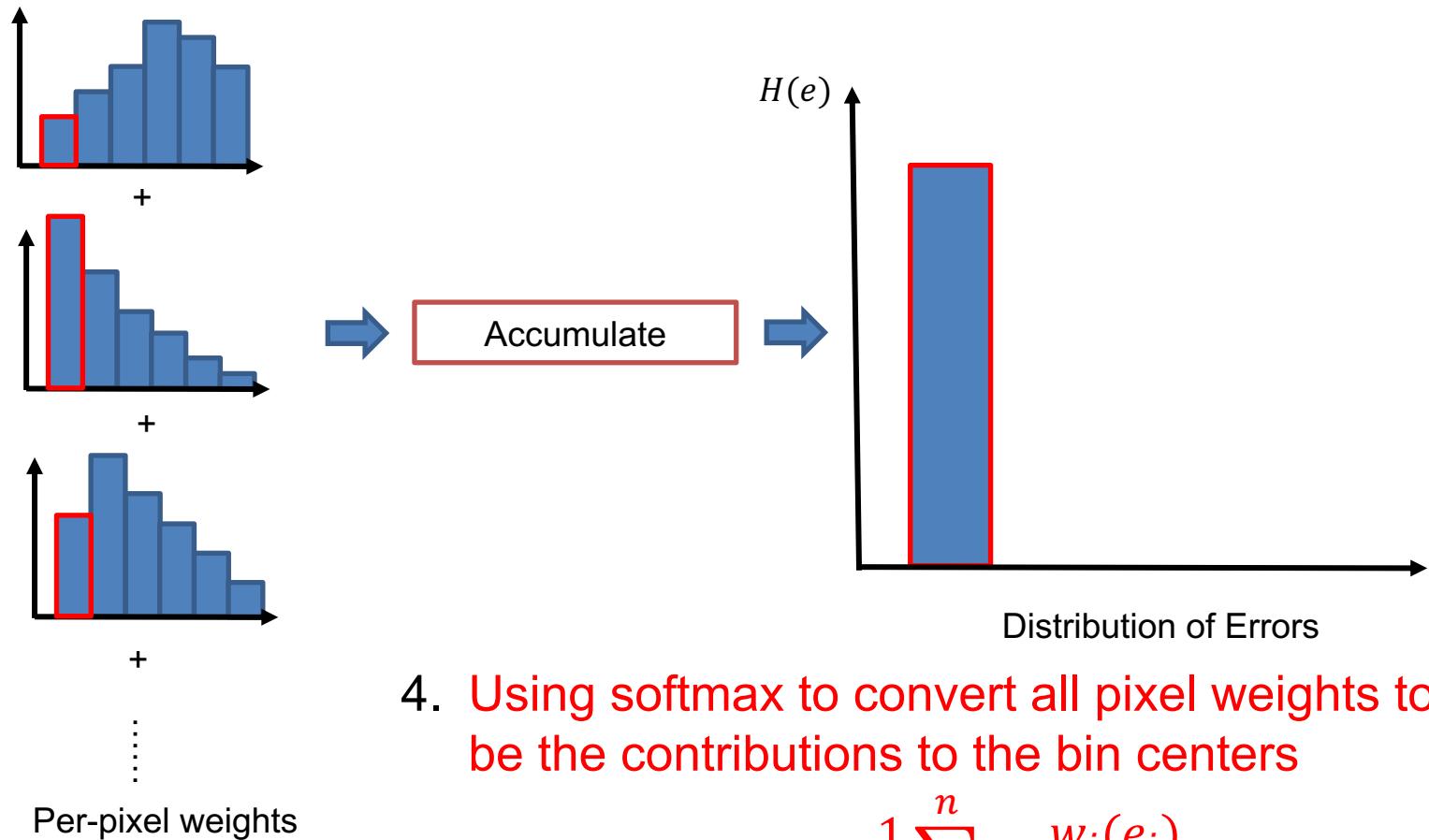
# Soft Histogramming



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2. Compute  $L2$  distances between  $e_i$  and bin centers:  
 $d_{i,1}, d_{i,2}, \dots, d_{i,m}$
3. Convert the distances  $d_{i,j}$  to pixel weights

$$w_j(e_i) = \lambda_1 \exp\left(-\frac{d_{i,m}}{\lambda_2}\right)$$

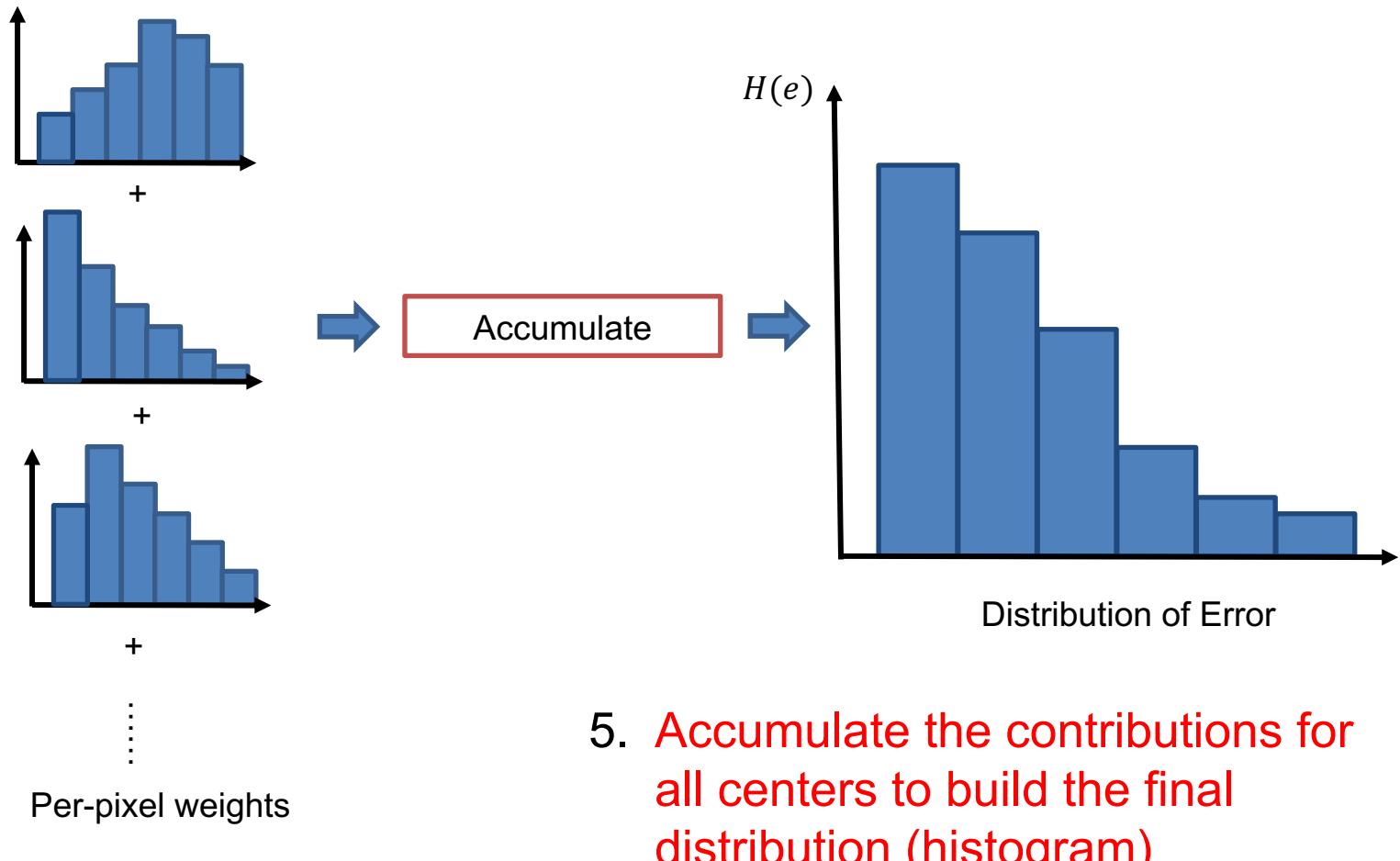
# Soft Histogramming



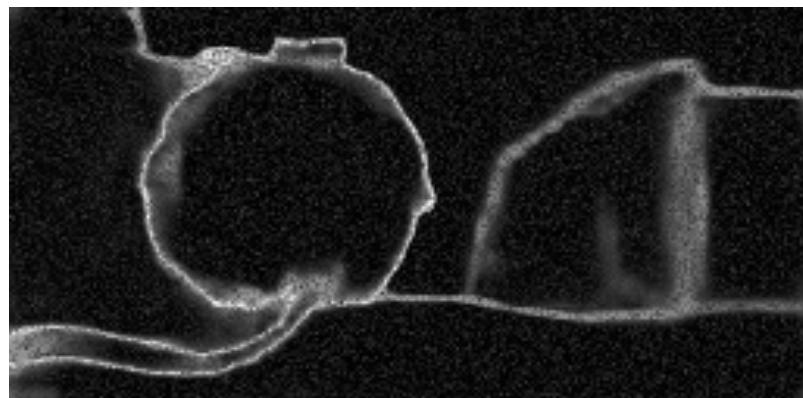
4. Using softmax to convert all pixel weights to be the contributions to the bin centers

$$H_j(e_i) = \frac{1}{n} \sum_{i=0}^n \frac{w_j(e_i)}{\sum_{j=m}^n w_j(e_i)}$$

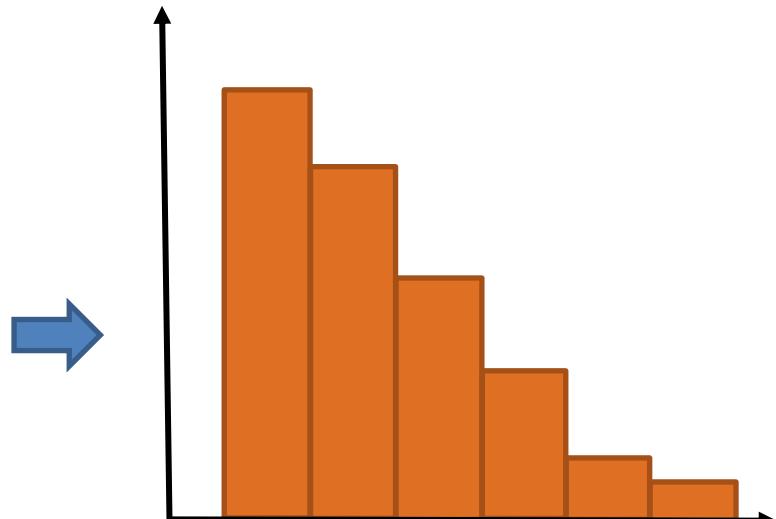
# Soft Histogramming



# Convert Uncertainty Map to Distribution



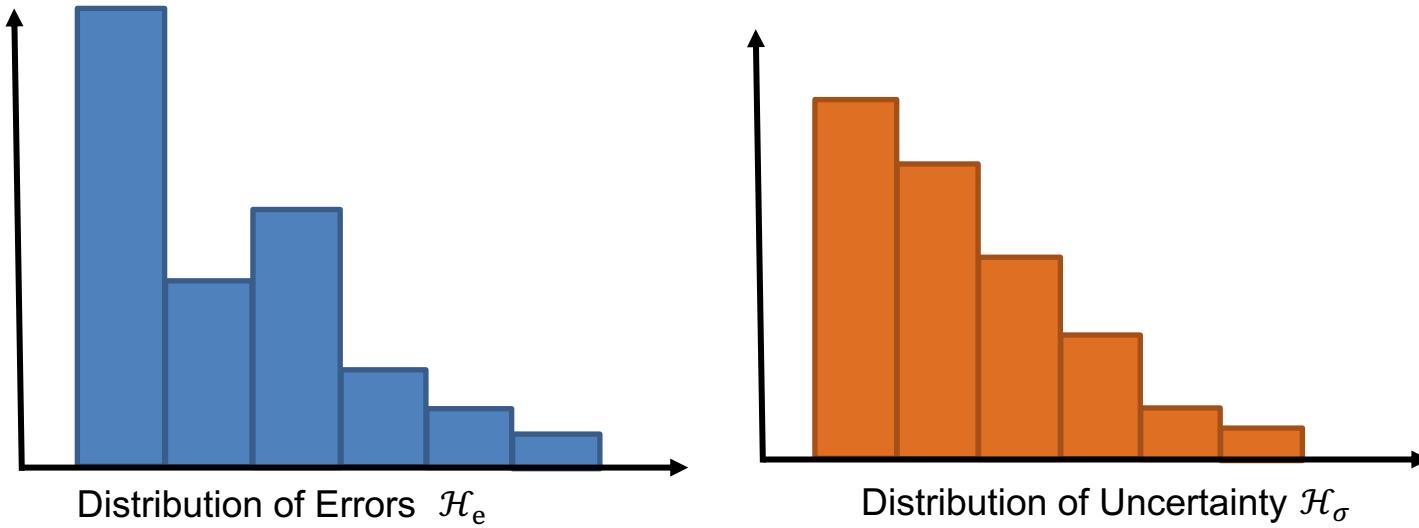
Uncertainty Map



Distribution of Uncertainty

Using **the same bin centers** as for the errors!

# Matching the Distribution of Error



Using KL-divergence loss  $\mathcal{L}_{KL} = \mathcal{D}_{KL}(\mathcal{H}_e || \mathcal{H}_\sigma)$

# Loss Function

$$\mathcal{L} = \sum_{k=1}^K c^k \cdot (\mathcal{L}_{log}^k + \mathcal{L}_{KL}^k)$$

- All disparity and uncertainty maps are **upsampled to the highest resolution**
- $c^k$  denotes **the coefficients** for the  $k^{\text{th}}$  resolution level  
*(We use  $K = 4$  in our experiments)*
- **Sum up** the two loss across all resolution

# Datasets & Baselines

## Datasets



Scene Flow



Virtual KITTI 2

Synthetic Data



DrivingStereo

Real Data

## Baselines

- Pick *strong baselines* according to recent survey (Poggi et al., 2021)
- GwcNet (Guo et al., 2019) + L1
- LAF-Net (Kim et al., 2019) + BCE [*Confidence Network*]
- GwcNet +  $\mathcal{L}_{log}$  (Kendall and Gal., 2017)

# Primary Results – Disparity & Uncertainty Estimation

*In domain:* train on VK2, test on VK2

Dataset	Method	EPE( $\downarrow$ )	MAPE( $\downarrow$ )	AUC Opt.( $\downarrow$ )	AUC Est.( $\downarrow$ )
Virtual KITTI 2	GwcNet + $\mathcal{L}_{log}$	0.3899	0.4136	4.6872	12.5320
	SEDNet	<b>0.3236</b>	<b>0.3561</b>	<b>4.2767</b>	<b>9.9843</b>

*Cross domain:* train on VK2, test on DS-Weather

Dataset	Method	EPE( $\downarrow$ )	MAPE( $\downarrow$ )	AUC Opt.( $\downarrow$ )	AUC Est.( $\downarrow$ )
DS-Weather	GwcNet + $\mathcal{L}_{log}$	2.3944	2.1443	41.1909	95.4264
	SEDNet	<b>1.7051</b>	<b>1.5842</b>	<b>39.8057</b>	<b>87.1882</b>

- **MAPE** is the mean L1 distance between the error and the observation uncertainty scalar,  $\sigma$ .
- Please see the paper for more results.

# Qualitative Results – Scene Flow

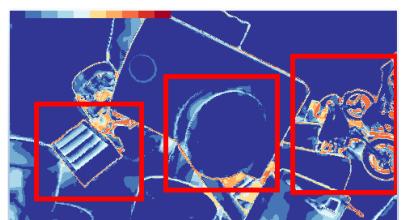
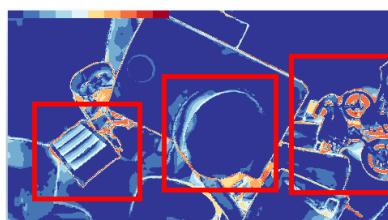
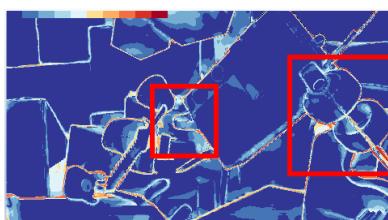
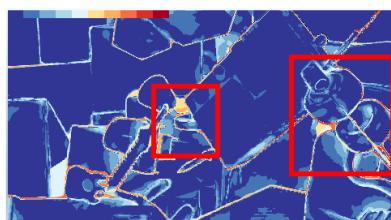
Synthetic stereo pairs for flying objects



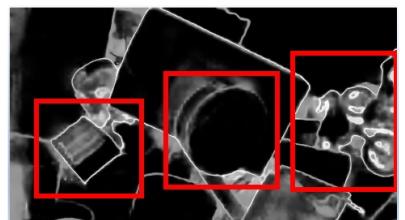
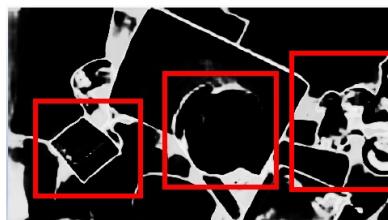
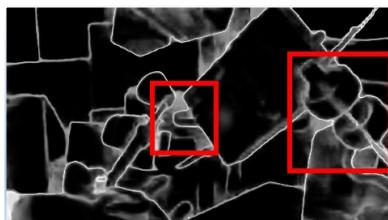
Disp



Error



Uncert



$\mathcal{L}_{log}$

SEDNet

$\mathcal{L}_{log}$

SEDNet

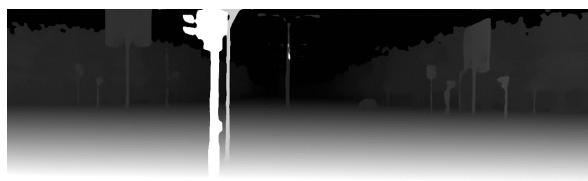
SEDNet has smaller errors and more accurate uncertainty.

# Qualitative Results – Virtual KITTI 2

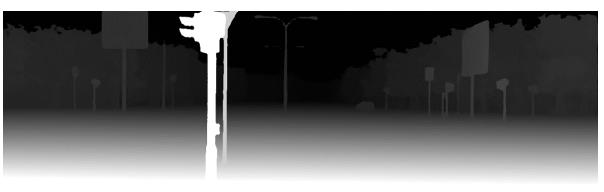
Synthetic stereo pairs for driving in different weather



Morning



Foggy



Rainy



Sunset



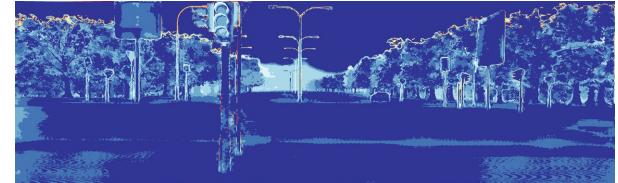
$\mathcal{L}_{log}$

SEDNet

SEDNet has better disparity estimation in different weather.

# Qualitative Results – Virtual KITTI 2

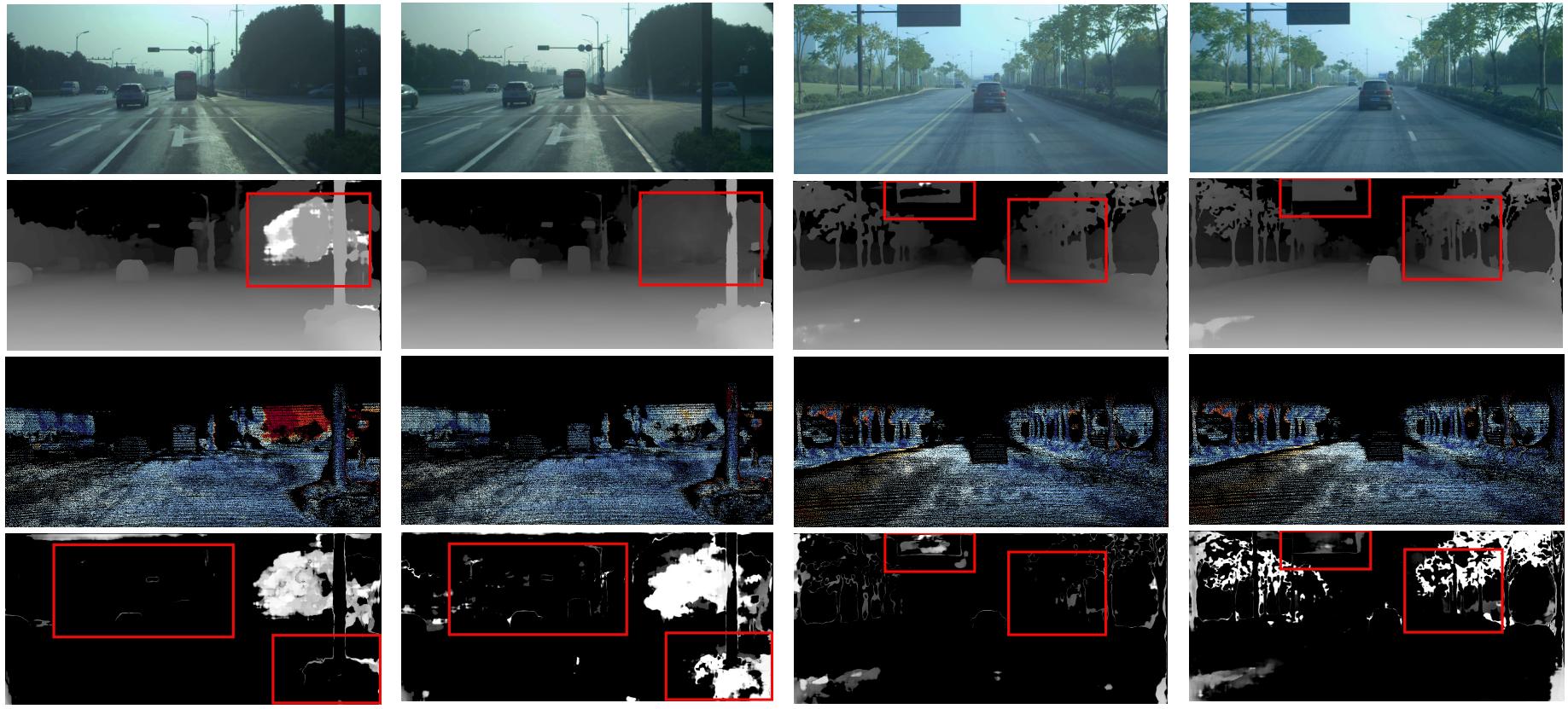
Comparing the challenging samples



The improvement of SEDNet on disparity and uncertainty estimation is more visible especially under bad weather.

# Qualitative Results – DrivingStereo

Real stereo pairs for foggy weather



$\mathcal{L}_{log}$

SEDNet

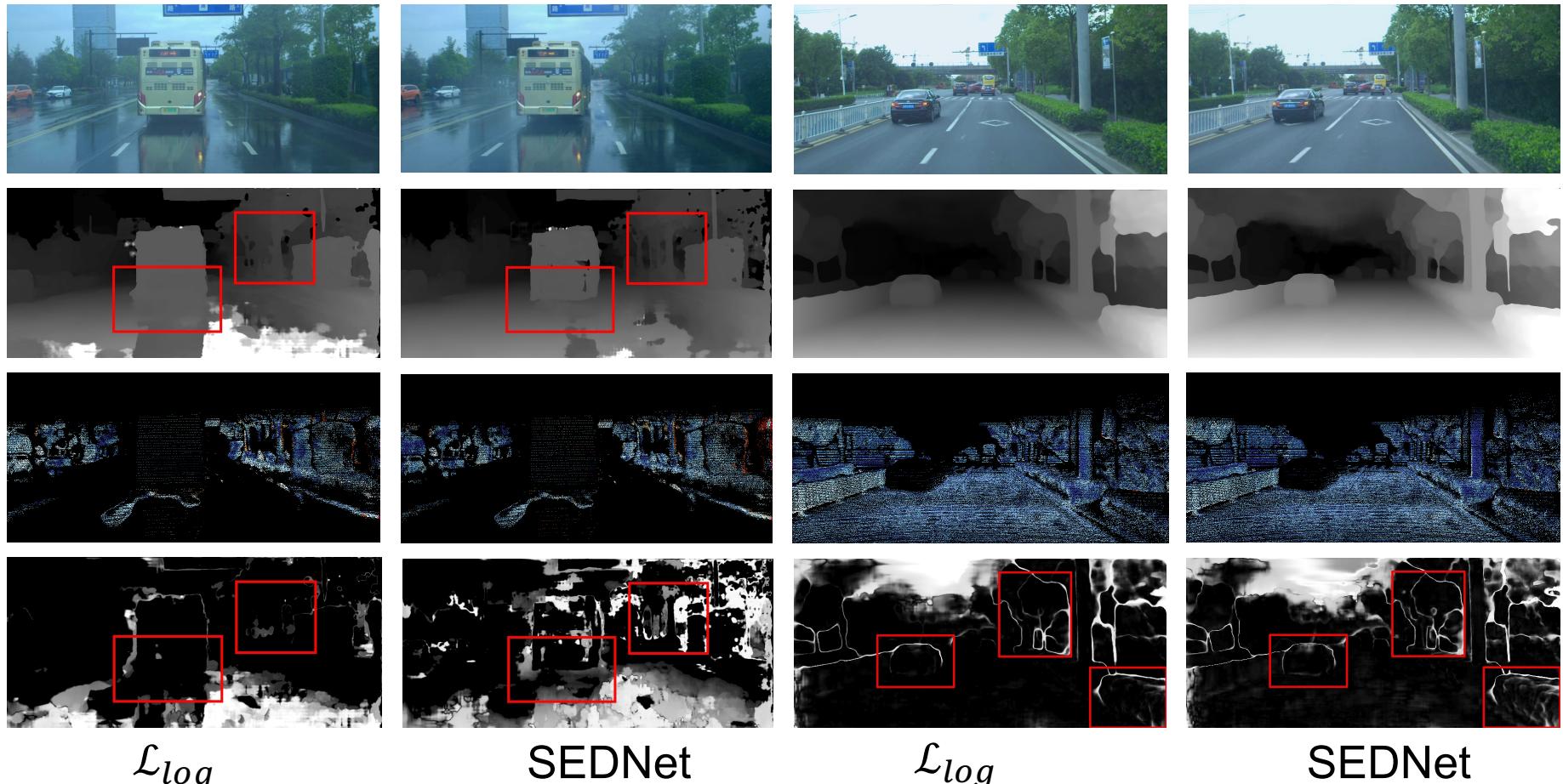
$\mathcal{L}_{log}$

SEDNet

- The foggy day are usually **very dark**, which makes distinguishing objects in the shadow difficult.
- SEDNet still performs better in predicting the uncertainty of the objects **far from the camera** and in the bottom right **dark corner**.

# Qualitative Results – DrivingStereo

Real stereo pairs for rainy weather



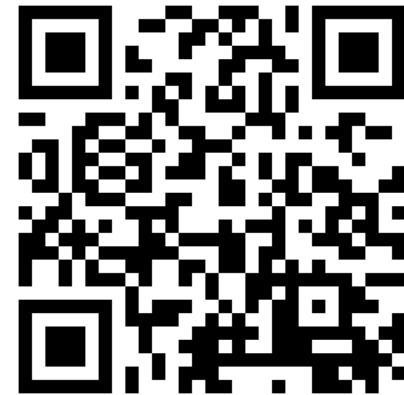
- Rainy images suffer from **poor illumination**, also face challenges due to **reflections in the water**.
- SEDNet captures **more faithful details** in both disparity and uncertainty maps.



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# Code available at

<https://github.com/lly00412/SEDNet>



Poster Section : THU-AM-072

## Thank you !