Learned trajectory embedding for subspace clustering

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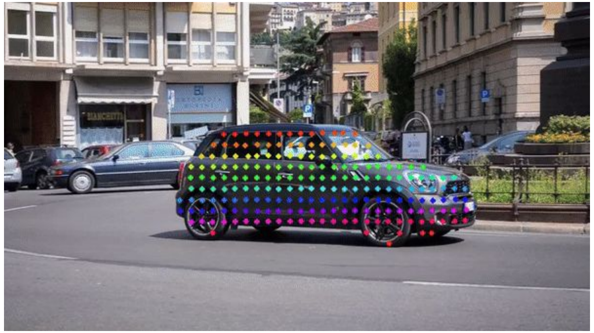
Point trajectories



Courtesy of **Tumanyan, Singer et al.** *DINO-tracker: taming DINO for self-supervised point tracking in a single video*

Courtesy of **Wang et al.** *Tracking everything everywhere all at once*

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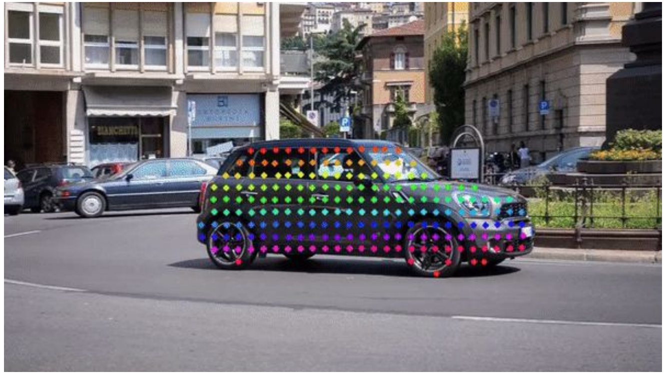
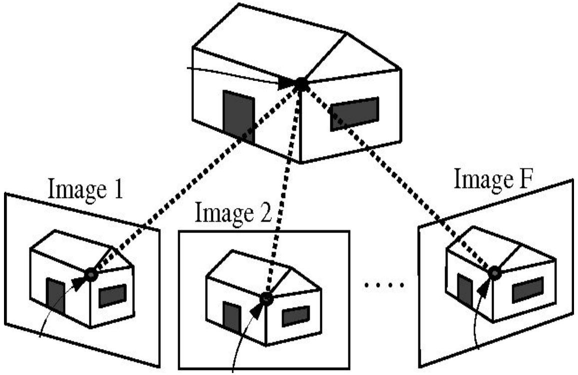
Rigid motion estimation

Matrix factorization for shape and motion reconstruction

$$\underbrace{\mathtt{M}}_{2F\times P} \approx \overbrace{\underbrace{\mathtt{B}}_{2F\times r}}^{\text{motions}} \overbrace{\underbrace{\mathtt{C}}_{r\times P}}^{\text{shapes}}, \qquad r \ll 2F$$





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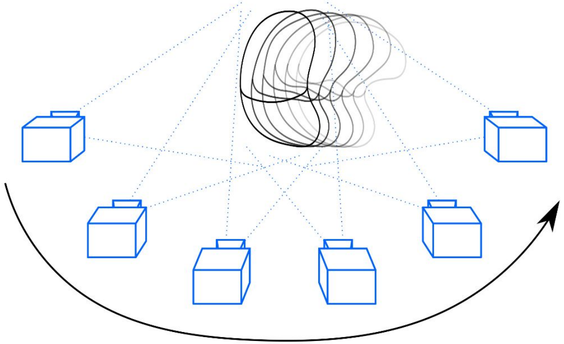
Nonrigid motion estimation

Matrix factorization for shape and motion reconstruction

$$\underbrace{\mathtt{M}}_{2F \times P} \approx \overbrace{\underbrace{\mathtt{B}}_{2F \times r}}^{\text{“motions”}} \overbrace{\underbrace{\mathtt{C}}_{r \times P}}^{\text{“shapes”}}, \qquad r \ll 2F$$



Courtesy of **Badias et al.** *MORPH-DSLAM: model order reduction for physics-based deformable SLAM*

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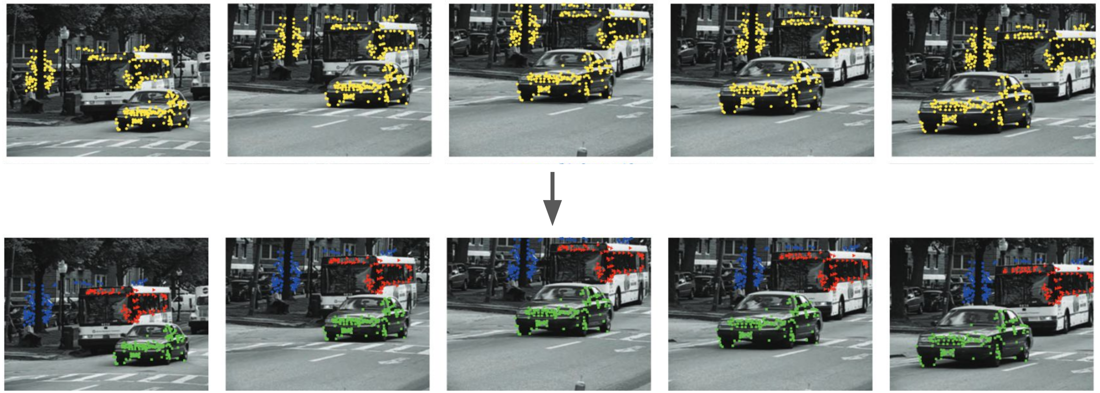
Motion segmentation

Multiple independent motions. Important for dynamic scene understanding

Chicken-and-egg problem (even for **rigid motion**) + contaminated by outliers and missing points

$$\mathrm{M}\,\underbrace{\mathrm{P}_{\pi}}_{P\times P} \approx \Big[\underbrace{\mathrm{B}_1\mathrm{C}_1}_{\text{group 1}} \quad \dots \quad \underbrace{\mathrm{B}_c\mathrm{C}_c}_{\text{group c}}\Big]$$



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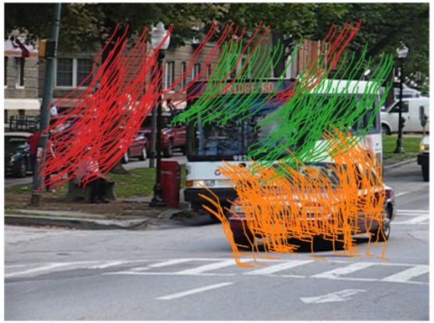
Motion segmentation methods

- Hypothesis generation-based *Robust statistical methods, joint optimization*
- Spectral clustering *Affinity matrix design, pairwise or multi-view relations*
  - Sparse subspace clustering *Use self-expressiveness, sparse optimization*

*“Unfortunately, traditional cluster-based trajectory segmentation methods rely on **heavy optimization** and **hand-crafted features**, and are hard to scale with dense trajectories”* — **Zhao et al.** ParticleSfM

Heavy test-time optimization



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Proposed method

Goal: learn trajectory feature representation useful for clustering
so that no simultaneous grouping and motion estimation at test-time is needed

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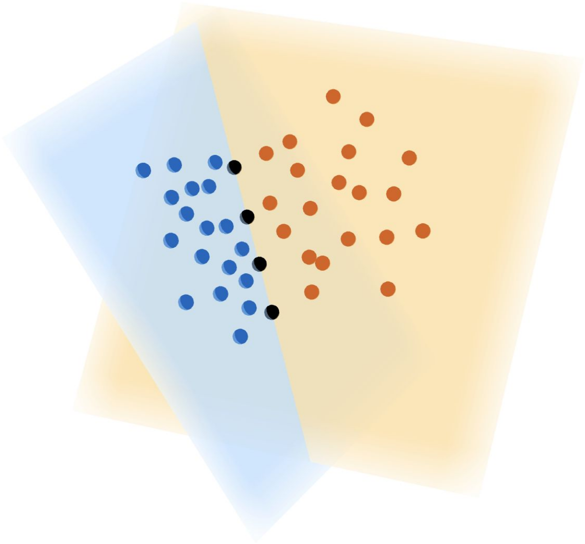
Disjoint subspace assumption

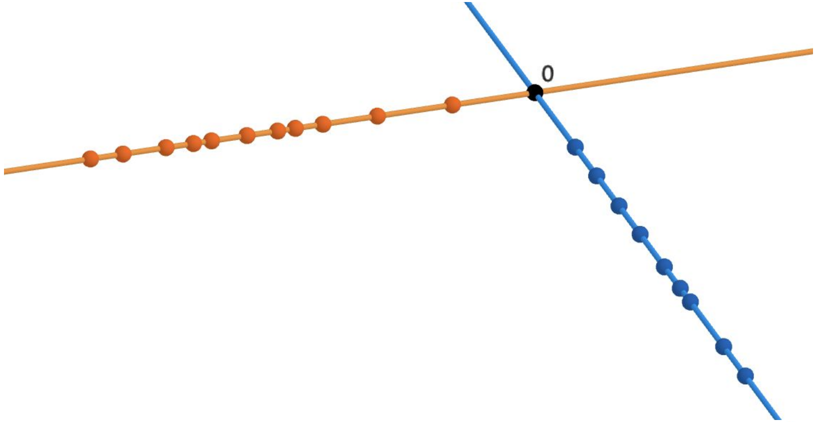
- Motion models do not intersect in high dimensional trajectory space
- In this work, we build on this **disjoint subspace assumption**

Planes in 3D



vs.

Motions as low dim subspaces in high dim space



$$\dim(\mathcal{S}_i \oplus \mathcal{S}_j) = \dim(\mathcal{S}_i) + \dim(\mathcal{S}_j)$$

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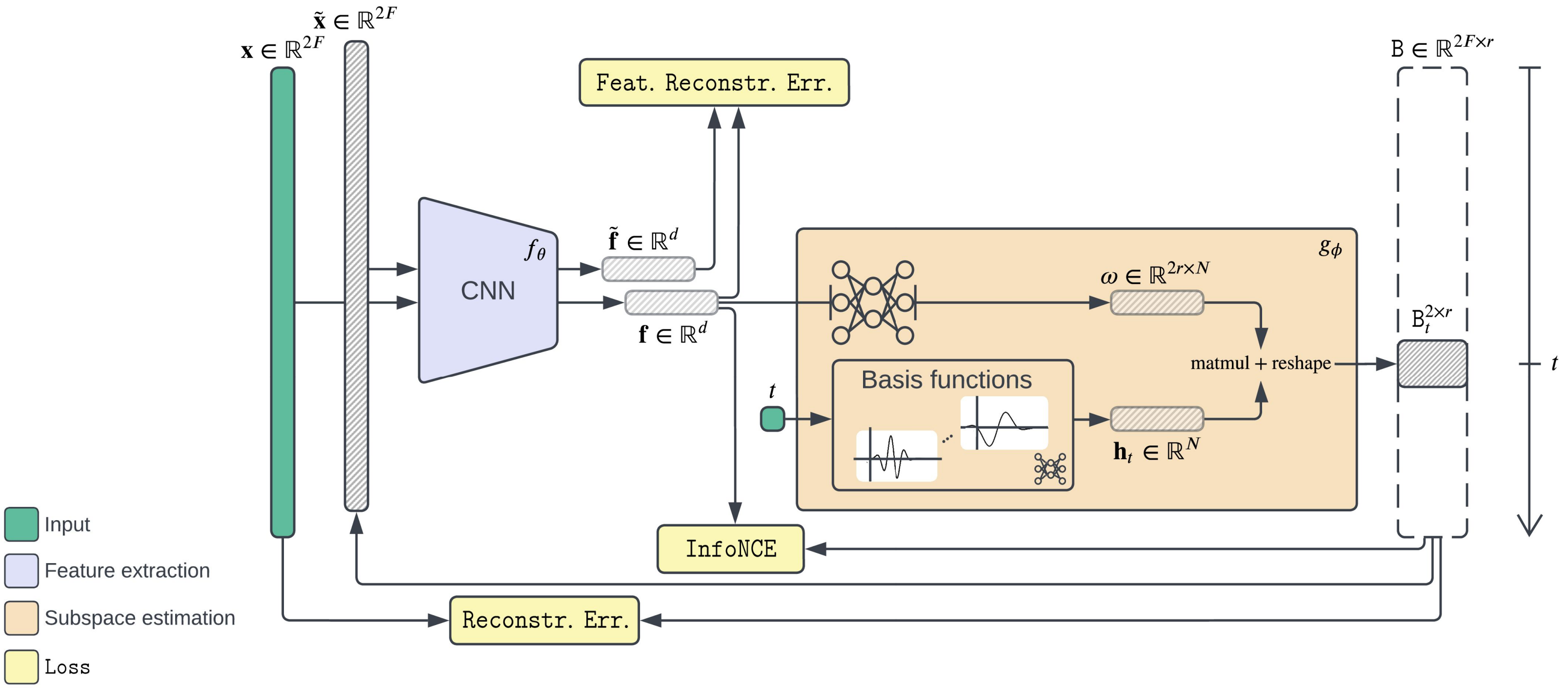
Proposed approach



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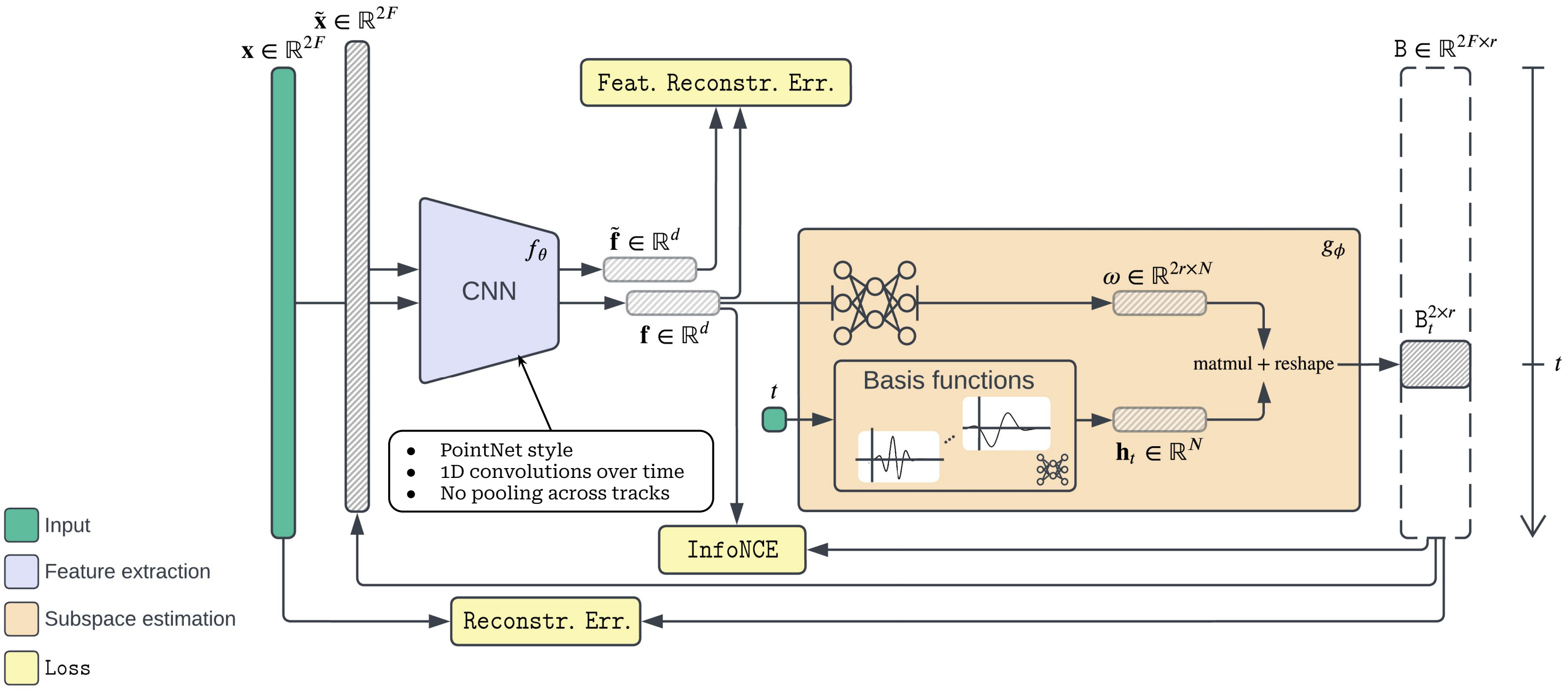

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WASP WALLENBERG AI, AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM

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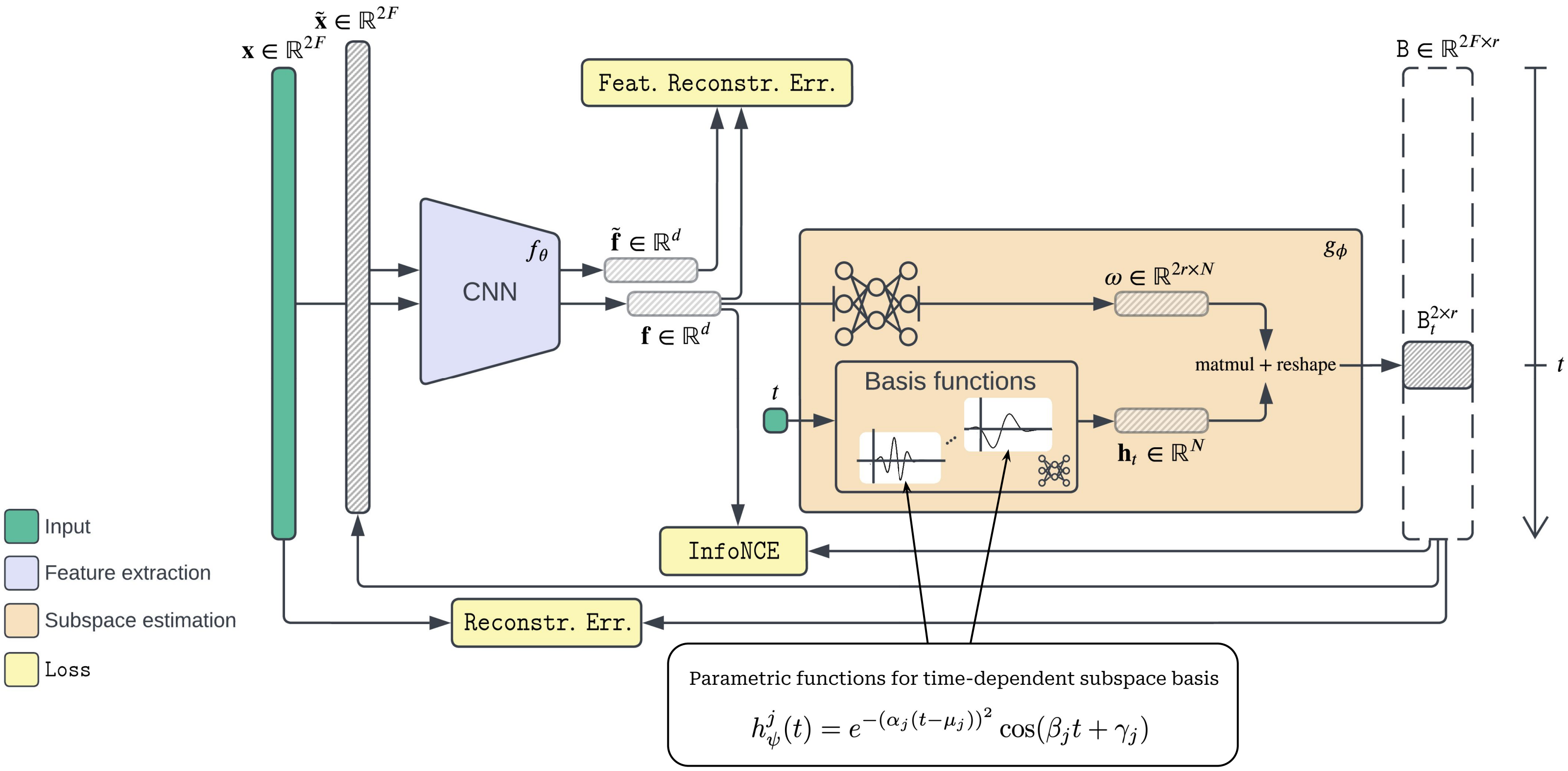

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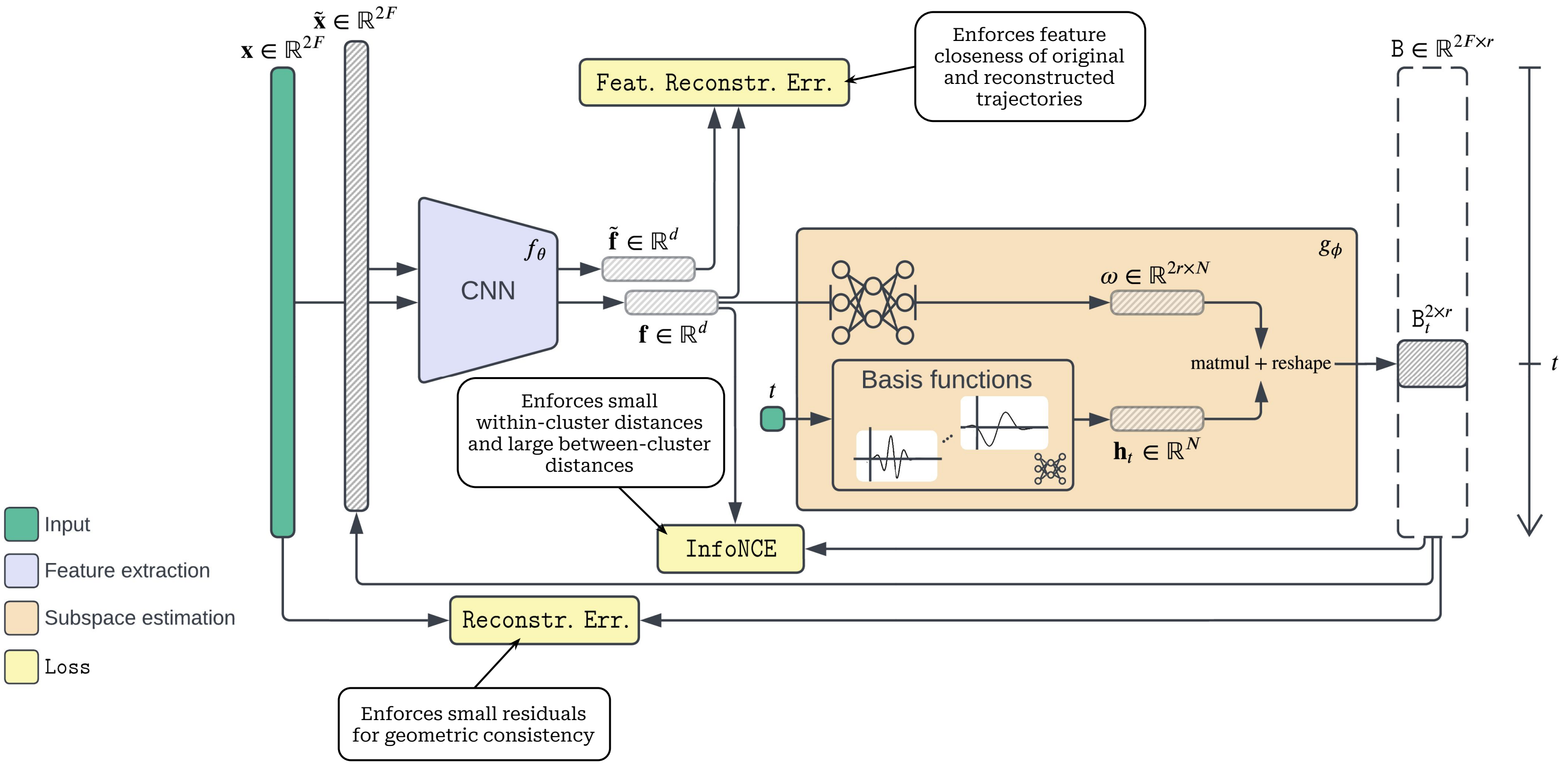

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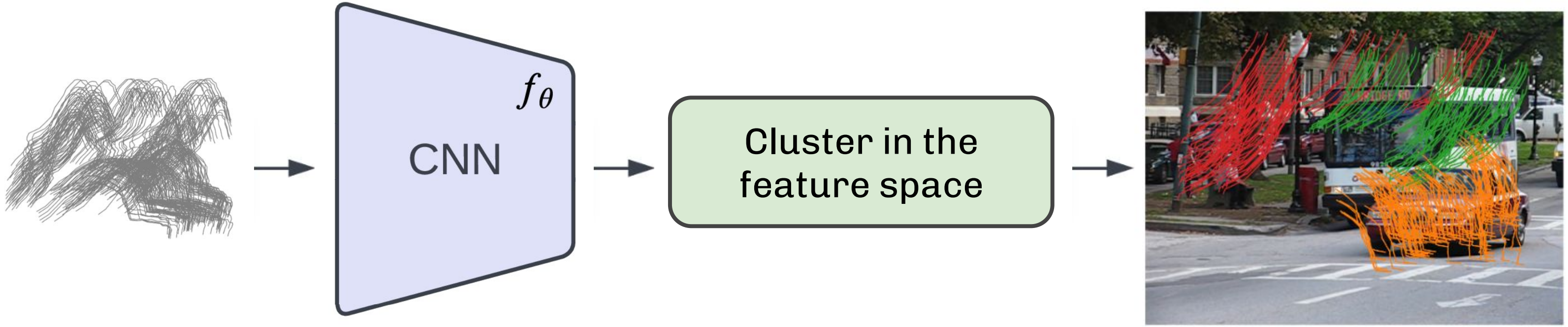
Inference: fully observed trajectory



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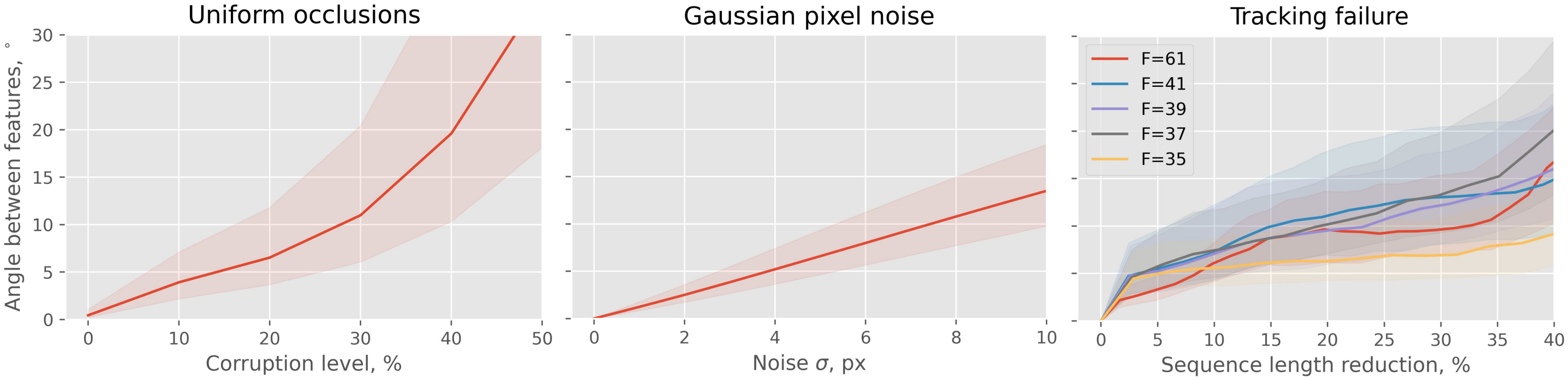
Approximate invariances of f_θ



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Inference: trajectory completion

- Formulate an objective to fill-in missing values

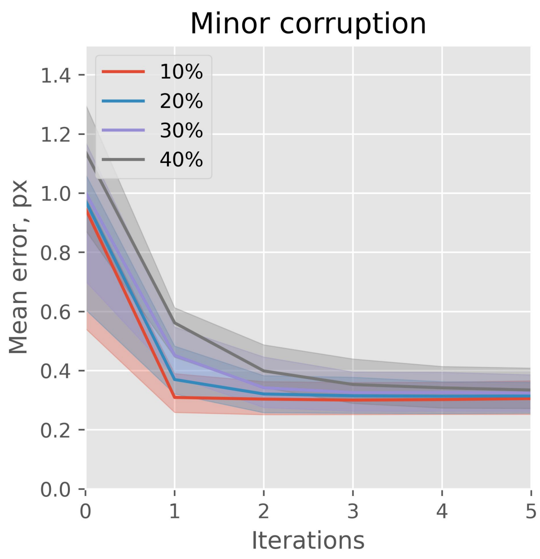
$$\left\|\hat{\mathbf{x}}(\bar{\mathbf{x}}) - \mathtt{B}\mathtt{B}^{\dagger}\hat{\mathbf{x}}(\bar{\mathbf{x}})\right\|^2 \rightarrow \min_{\bar{\mathbf{x}}}$$

- Obtain linear solution for a fixed subspace

$$\bar{\mathbf{x}}^* = \mathtt{A}(\mathtt{B})\mathbf{x}$$

- Yields an iterative procedure

$$\begin{aligned}\mathtt{B}_0 &\leftarrow B_{\theta,\phi}(\mathbf{x}_{\mathsf{vis}}, \mathbf{t})\\ \bar{\mathbf{x}}_i &\leftarrow \mathtt{A}(\mathtt{B}_{i-1})\mathbf{x}\\ \mathtt{B}_i &\leftarrow B_{\theta,\phi}(\mathbf{w}\odot\mathbf{x}+\bar{\mathbf{w}}\odot\bar{\mathbf{x}}_i, \mathbf{t})\end{aligned}$$



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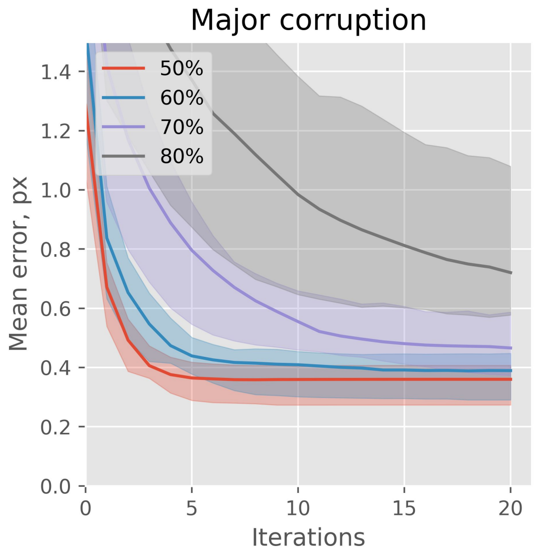

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Clustering error on standard datasets

Method	Hopkins155 Mean	Hopkins155 Median	Hopkins155 Time	Hopkins12 Mean	Hopkins12 Median	KT3DMoSeg Mean	KT3DMoSeg Median
RANSAC	9.76	3.21	194ms	-	-	-	-
GPCA	10.34	2.54	417ms	-	-	34.60	33.95
MSL	5.03	0.00	19h 11m	-	-	-	-
LSA	4.94	0.90	9.47s	-	-	38.30	38.58
ALC_5	3.76	0.26	5m 15s	3.81	0.17	24.31	19.04
$\mathrm{ALC}_{\mathrm{sp}}$	3.37	0.49	6m 11s	1.28	1.07	-	-
LRR	5.41	0.53	1.1s	-	-	33.67	36.01
SSC	2.45	0.20	920ms	-	-	33.88	33.54
RSIM	1.01	0.00	176ms	0.68	0.70	-	-
MultiCons	4.40	-	40ms	-	-	-	-
Ours	0.62	0.0	9ms	5.12	2.04	5.85	0.80