FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
Comenius University
Bratislava

FACULTY OF ELECTRICAL ENGINEERING
CTU IN PRAGUE

Robust Self-calibration of Focal Lengths from the Fundamental Matrix

Viktor Kocur, Daniel Kyselica, Zuzana Kúkelová

TERAIS

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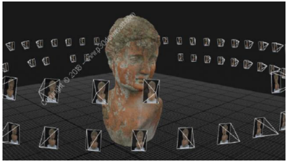
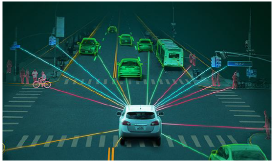
3D Vision Applications



3D Reconstruction



Autonomous Driving

Augmented Reality

SLAM and Localization

- calibrate cameras
- estimate camera poses/motion
- triangulate points

Camera Calibration

Offline Calibration

Self-calibration
(autocalibration)

Camera Calibration

Offline Calibration

☺ Can provide complex camera model

☹ Not always possible

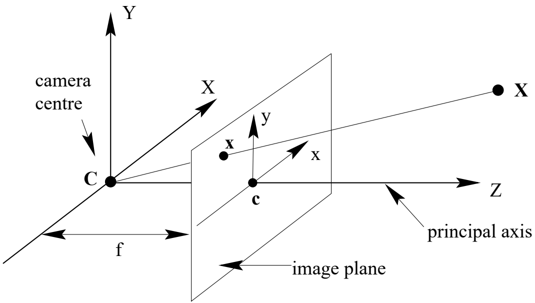
Self-calibration

(autocalibration)

☺ Can be used "in the wild"

☹ Usually simpler camera models

Image adopted from: Zuzana Kukelova et al. “Efficient solution to the epipolar geometry for radially distorted cameras.” *CVPR*. 2015



We want camera intrinsics: focal length, principal point, skew, distortion coeffs...

We can often use additional assumptions: no skew, no distortion, principal point in the image center, square pixels $\rightarrow$ only f remains unknown.

Image adopted from: R. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*. 2003

Camera Focal Length Estimation

Possible from a single image in some cases:

- Known or learned objects/scene
- Regular geometry observed, e.g.:
  - Calibration patterns
  - Repeating patterns
  - Orthogonal sets of parallel lines

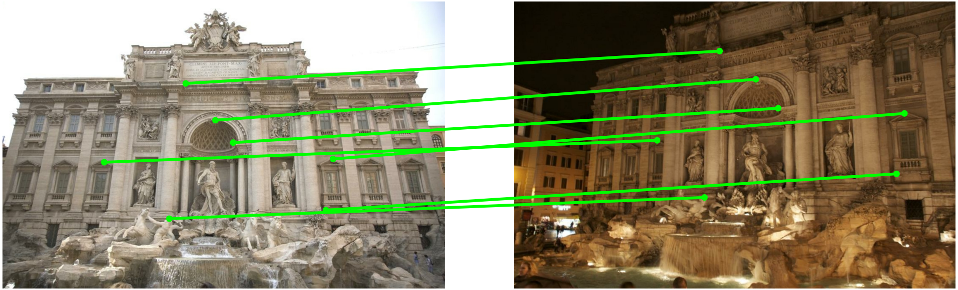
In general we need at least two-views!

Fundamental Matrix

$$\mathbf{F} = \mathbf{K}_2^{-\top}[\boldsymbol{t}]_\times \mathbf{R}\mathbf{K}_1^{-1}$$

$\mathbf{F}$ contains information on camera intrinsics $\mathbf{K}_i = \begin{bmatrix} f_i & 0 & c_{x,i} \\ 0 & f_i & c_{y,i} \\ 0 & 0 & 1 \end{bmatrix}$ and pose $(\mathbf{R}, \boldsymbol{t})$.

Fundamental Matrix



$$\mathbf{F} = \mathbf{K}_2^{-\top} [\boldsymbol{t}]_\times \mathbf{R} \mathbf{K}_1^{-1}$$

We estimate **F** using point correspondences and RANSAC

Bougnoux Formula

We can usually assume that the principal points ($\boldsymbol{c}_1, \boldsymbol{c}_2$) are in the image centers and estimate the focal length of the first camera f_1 from $\mathbf{F}$ using:[3]

$$f_1^2 = -\frac{\boldsymbol{c}_2^\top [\boldsymbol{e}_2]_\times \hat{\mathbf{I}} \mathbf{F} \boldsymbol{c}_1 \boldsymbol{c}_1^\top \mathbf{F}^\top \boldsymbol{c}_2}{\boldsymbol{c}_2 [\boldsymbol{e}_2]_\times \hat{\mathbf{I}} \mathbf{F} \hat{\mathbf{I}} \mathbf{F}^\top \boldsymbol{c}_2} \tag{1}$$

Equivalent formula exists for f_2^2.

Both can also be calculated directly from elements of $\mathbf{F}$.[4]

[3] Sylvain Bougnoux. “From projective to euclidean space under any practical situation, a criticism of self-calibration.” *ICCV*. 1998

[4] Oleh Rybkin. “Robust Focal Length Estimation.” Supervised by: Tomáš Pajdla. Bachelor’s Thesis. Czech Technical University in Prague, 2017

Problems

Singularity: The denominator vanishes when the principal axes of the two cameras are coplanar (they intersect). This makes the formula unstable in many practical situations!

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Imaginary f: The formula provides f_i in squared form, thus sometimes yielding imaginary f_i. This can be caused by instability or errors in $\mathbf{F}, \boldsymbol{c}$.

Alternative - Iterative Optimization

Hatley and Silpa-Anan[5] propose to optimize $\mathbf{F}, \boldsymbol{c}_1, \boldsymbol{c}_2$ with LM w.r.t cost:

$$C = w_{\mathbf{F}} C_{\mathbf{F}} + \sum_{i \in \{1,2\}} w_{f_{i,p}} (f_{i,p}^2 - f_i^2)^2 + w_{\boldsymbol{c}_{i,p}} ||\boldsymbol{c}_{i,p} - \boldsymbol{c}_i||^2 + w_p C_p(f_i) \tag{2}$$

- $w_{\mathbf{F}} C_{\mathbf{F}}$: Sampson Error w.r.t. inlier matches
- $w_{f_{i,p}} (f_{i,p}^2 - f_i^2)^2$: Prior for f_i
- $w_{\boldsymbol{c}_{i,p}} ||\boldsymbol{c}_{i,p} - \boldsymbol{c}_i||^2$: Prior for $\boldsymbol{c}_i$
- $w_p C_p(f_i)$: Penalty for low f_i

[5] Richard Hartley and Chanop Silpa-Anan. “Reconstruction from two views using approximate calibration.” *ACCV*. 2002

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- ☺ Able to correct for error in $\boldsymbol{c}_1, \boldsymbol{c}_2, \mathbf{F}$
- ☺ Outputs real f_1, f_2

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☺ Able to correct for error in $\boldsymbol{c}_1, \boldsymbol{c}_2, \mathbf{F}$

☺ Outputs real f_1, f_2

☹ f_i still from eq. (1) → singularities

☹ Sampson Error is expensive

☹ Many iterations to converge

☹ Weights need to be tuned

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Our Contribution

- New method for self-calibration of f_1, f_2 from $\mathbf{F}$
  - Solved as optimization problem using a simpler cost function
  - We propose an efficient iterative algoritm for optimization of the cost function
  - More robust (accurate) than SOTA
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- New RANSAC check $\rightarrow$ further precision improvement and faster RANSAC
- Both extensively tested on different real-world datasets with different features and feature matchers and RANSAC variants

Estimating f_i - Our Approach

We solve an optimization problem:

$$\min_{f_1,f_2,\mathbf{c}_1,\mathbf{c}_2} \sum_{i=1,2} w_i^f (f_i - f_i^p)^2 + w_i^{\mathbf{c}} \|\mathbf{c}_i - \mathbf{c}_i^p\|^2 \tag{3}$$

[6] Manolis IA Lourakis and Rachid Deriche. “Camera self-calibration using the Kruppa equations and the SVD of the fundamental matrix: The case of varying intrinsic parameters.” PhD thesis. INRIA, 2000

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$$\text{s.t.} \quad \kappa_1 = \sigma_1(\mathbf{v}_1^\top \omega_1^* \mathbf{v}_1)(\mathbf{u}_1^\top \omega_2^* \mathbf{u}_2) + \sigma_2(\mathbf{v}_1^\top \omega_1^* \mathbf{v}_2)(\mathbf{u}_2^\top \omega_2^* \mathbf{u}_2) = 0 \tag{4}$$

$$\kappa_2 = \sigma_1(\mathbf{v}_1^\top \omega_1^* \mathbf{v}_2)(\mathbf{u}_1^\top \omega_2^* \mathbf{u}_1) + \sigma_2(\mathbf{v}_2^\top \omega_1^* \mathbf{v}_2)(\mathbf{u}_1^\top \omega_2^* \mathbf{u}_2) = 0, \tag{5}$$

where κ_1, κ_2 are two Kruppa equations[6] derived from $\omega_i^* = \mathbf{K}_i \mathbf{K}_i^\top$ and SVD of $\mathbf{F}$.

[6] Manolis IA Lourakis and Rachid Deriche. "Camera self-calibration using the Kruppa equations and the SVD of the fundamental matrix: The case of varying intrinsic parameters." PhD thesis. INRIA, 2000

To solve the problem we use the Lagrangian.

$$L = \sum_{i=1,2} w_i^f (f_i - f_i^p)^2 + w_i^{\mathbf{c}} \|\mathbf{c}_i - \mathbf{c}_i^p\|^2 - 2\lambda_1\kappa_1 - 2\lambda_2\kappa_2 \quad (6)$$

Solving $\nabla L = \mathbf{0}$ using algebraic methods is not feasible - system too complex

[7] Peter Lindstrom. “Triangulation made easy.” *CVPR*. 2010

[8] Viktor Larsson, Kalle Astrom, and Magnus Oskarsson. “Efficient solvers for minimal problems by syzygy-based reduction.” *CVPR*. 2017

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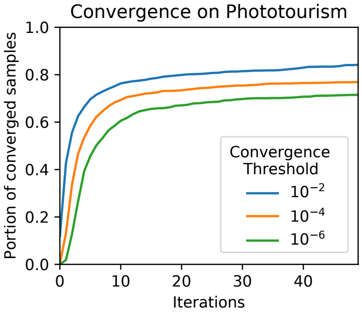

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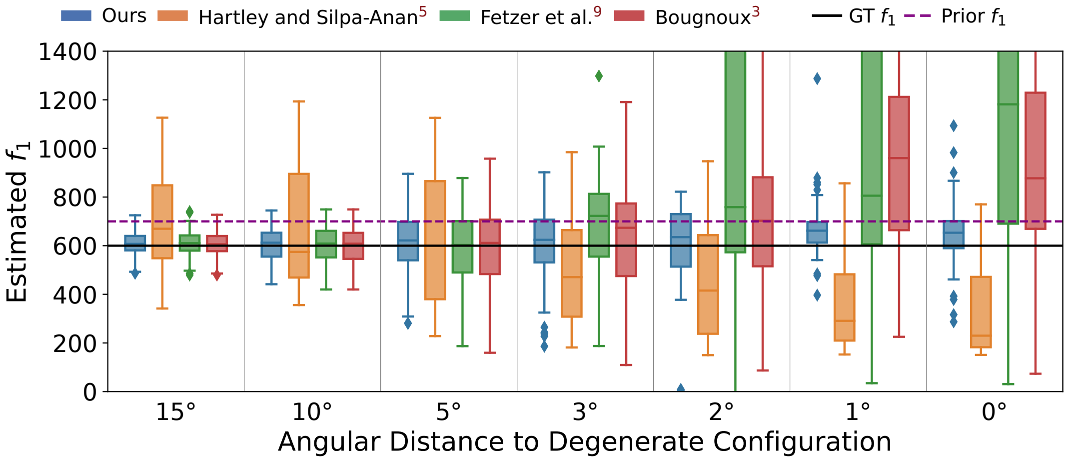
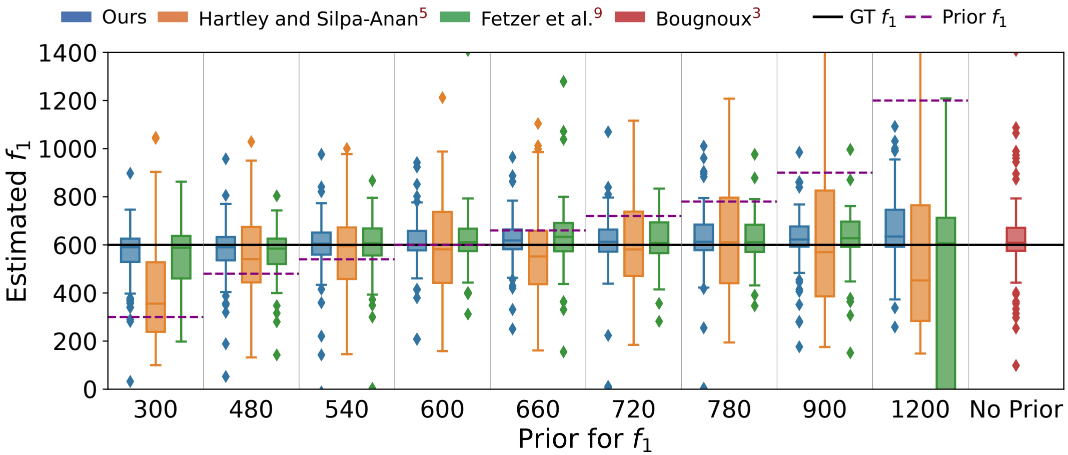
Synth Data - Performance in Degenerate Configurations



[9] Torben Fetzer, Gerd Reis, and Didier Stricker. "Stable intrinsic auto-calibration from fundamental matrices of devices with uncorrelated camera parameters." *CVPR*. 2020

Synth Data - Priors



Our method works with inaccurate priors!

Real Focal Length Checking (RFC)

F with imaginary f_i from the Bougnoux formula usually lead to fewer inliers than **F** with real f_i.

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[10] Yuhe Jin et al. “Image matching across wide baselines: From paper to practice.” *IJCV* (2021)

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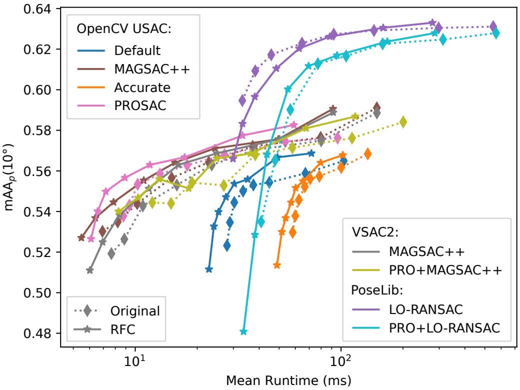
Results on real data[10] with LoFTR[11] →



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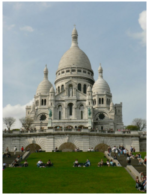
Real Data Evaluation

Evaluated on two large-scale datasets with COLMAP GT:

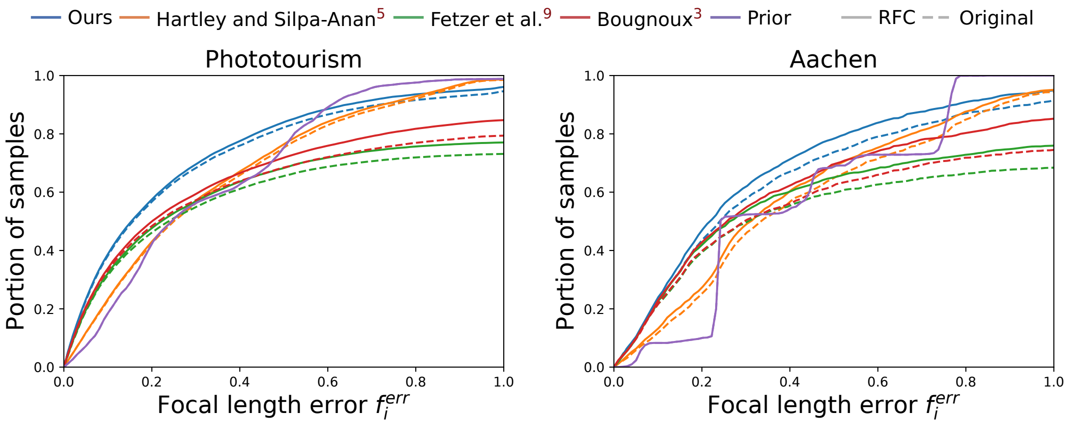
- Phototourism[10]
  - 12 evaluation scenes of historical landmarks
  - 1000 image pairs per scene
- Aachen Day-Night v1.1[12]
  - 5000 image pairs

We use COLMAP priors: $f_i^p = 1.2 \max(width_i, height_i)$



[12] Zichao Zhang, Torsten Sattler, and Davide Scaramuzza. "Reference pose generation for long-term visual localization via learned features and view synthesis." *IJCV* (2021)

f_i Estimation Accuracy



Our iterative method and RFC beat SOTA in terms of $f_i^{err} = \frac{|f_i^{est} - f_i^{gt}|}{f_i^{gt}}$.

F estimated using Magsac++[13] + LoFTR[11] matches - more experiments in paper.

[13] Daniel Barath et al. “MAGSAC++, a fast, reliable and accurate robust estimator.” *CVPR*. 2020

Pose Estimation Accuracy

		Phototourism[10]			Aachen Day-Night v1.1 [12]		
Method	RFC	Median p_{err}	mAA_p 10°	mAA_p 20°	Median p_{err}	mAA_p 10°	mAA_p 20°
Ours		6.43°	40.48	56.01	9.52°	27.62	46.28
Ours	✓	**6.29°**	**41.03**	**56.78**	**8.78°**	**29.29**	**48.13**
Hartley[5]		9.19°	30.15	46.94	12.19°	21.10	38.74
Hartley[5]	✓	9.00°	30.61	47.69	11.37°	22.12	40.61
Fetzer[9]		9.40°	33.00	47.25	12.33°	24.34	39.69
Fetzer[9]	✓	8.94°	33.64	48.51	10.66°	25.62	42.16
Bougnoux[3]		7.55°	37.17	52.70	10.25°	26.73	45.07
Bougnoux[3]	✓	7.39°	37.57	53.21	9.69°	27.25	45.61
Prior		11.17°	22.63	41.98	13.00°	17.06	37.27
Prior	✓	11.05°	22.73	42.31	12.73°	17.68	38.05

Pose error $p_{err} = \max\left(\angle\left(\mathbf{R}^{est}, \mathbf{R}^{gt}\right), \angle\left(\boldsymbol{t}^{est}, \boldsymbol{t}^{gt}\right)\right)$ and mean average accuracy (mAA_p) evaluation shows that our method leads to better poses.

Summary

- Our proposed method for robust self-calibration of f from **F** beats SOTA in terms of accuracy and is faster than previous iterative approaches.
- If you plan to use images with unknown intrinsics (crowdsourced, changing f of a single camera) consider using our method.
- If you need to estimate **F** with RANSAC consider using RFC.
- More info (method details, experiments - case of $f_1 = f_2$) in the paper.

Code available on Github.

Thank you for your attention!