Circuit Design and Efficient Simulation of Quantum Inner Product and Empirical Studies of Its Effect on Near-Term Hybrid Quantum-Classic Machine Learning

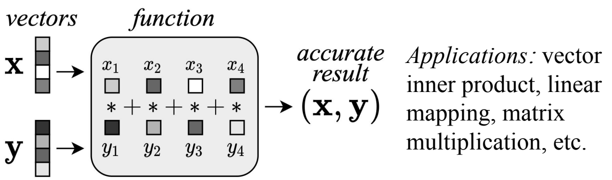
Hao Xiong*, YehuiTang*, Xinyu Ye*, Junchi Yan+
{taxuexh, yehuitang, xinyu_ye, yanjunchi}@sjtu.edu.cn, Shanghai Jiao Tong University, *equal contribution, +corresponding authors

project page: https://github.com/ShawXh/qip_cvpr24

CVPR SEATTLE, WA JUNE 17-21, 2024

1 Introduction

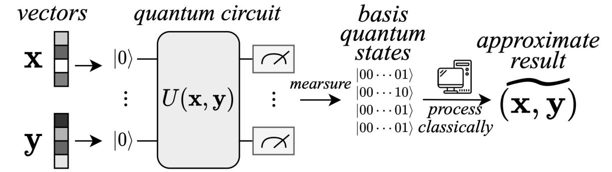
Inner product compuation



Applications: vector inner product, linear mapping, matrix multiplication, etc.

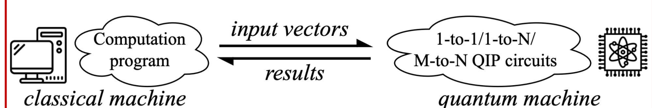
a) Classical computing

Quantum inner product compuation



b) Quantum computing

Quantum machine computes inner products by measuring the output quantum states of a designed quantum circuits



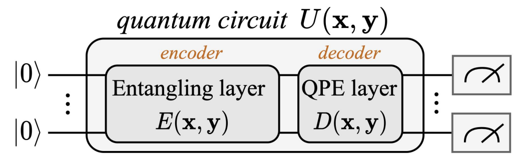
c) Hybrid quantum-classical computing

Classical machines upload vectors to the quantum machine, which computes inner products and return results to classical machines.

Contributions of the paper:

- Circuit Design of QIP on Quantum Computers.
- Efficient Simulation of QIP on Classic Computers.
- Evaluating QIP in ML on Classic Computers.

2 Quantum Circuit Design



a) Quantum circuits for QIP

Theorem 1 (1-to-1 QIP Circuit (for vector inner product)**).** *There exists a quantum circuit $U(\mathbf{x}, \mathbf{y})$ computing the inner product of two normalized vector $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ with complexity $O(\frac{\log d}{\epsilon})$ where ϵ is a given precision parameter.*

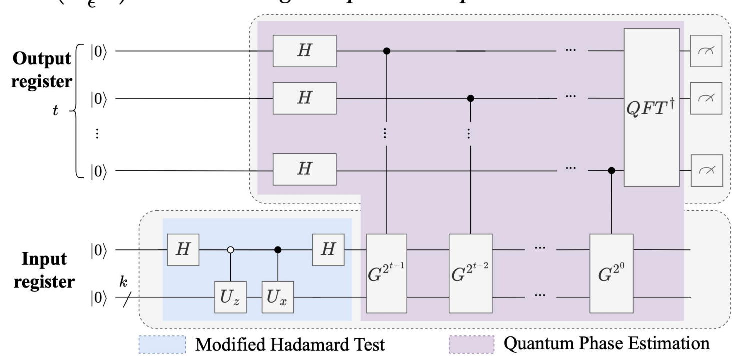


Figure 8. Quantum circuit of the 1-to-1 quantum inner product estimation.

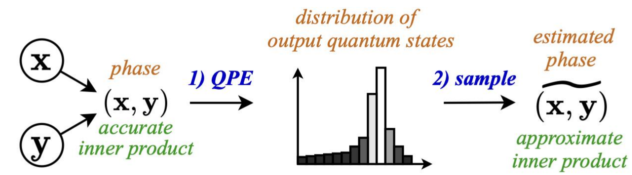
(For details please refer to the paper) **Theorem 2** for 1-to-N QIP Circuit (for linear mapping) and **Theorem 3** for M-to-N QIP Circuit (for matrix multiplication.

Time complexity of the QIP circuit: $O(\epsilon^{-1} \log MN \log d)$

Time complexity of classical computation: $O(MNd)$

Mode	Number of required qubits	Use case
1-to-1	$1 + t + \lceil \log d \rceil$	vector inner product
1-to-N	$1 + Nt + \lceil \log N \rceil + \lceil \log d \rceil$	linear mapping
M-to-N	$1 + MNt + \lceil \log N \rceil + \lceil \log M \rceil + \lceil \log d \rceil$	matrix multiplication

3 Fast Simulation



b) Fast simulation of the circuits for QIP

Idea: directly simulate the output quantum states by the circuits, but not the compuatational process of the circuits

Space omplexity of our simulation scheme: $\approx O(2^t)$

Space omplexity by circuit simulator (e.g. qiskit): $O(2^{2t})$

4 Selected Experiments

Experiment 1: ProjUNN with QIP, on MNIST

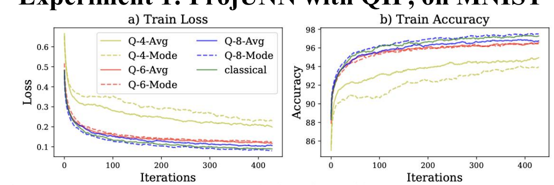


Figure 4. Convergence. 'Q': quantum method; '4/6/8': qubit # t; 'Avg/Mode': output strategy.

Table 6. Test accuracy after 20 epochs on MNIST.

	Accuracy (%)
Q-4-Avg	93.70
Q-4-Mode	92.96
Q-6-Avg	94.65
Q-6-Mode	95.37
Q-8-Avg	95.54
Q-8-Mode	**96.13**
classical	95.97

Experiment 2: Accuracy and efficiency comparison of the circuit simulator implemented in qiskit and our state simulator

	Simulator	MSE ↓	MAE ↓	Running Time (s) ↓
$d = 4$	Circuit (qiskit)	4.31e-4	1.63e-2	7.072
	State (ours)	numerically the same		0.036, **196x faster**
$d = 16$	Circuit (qiskit)	7.64e-4	2.32e-2	114.502
	State (ours)	numerically the same		0.037, **3095x faster**
$d = 64$	Circuit (qiskit)	9.05e-4	2.53e-2	2388.785
	State (ours)	numerically the same		0.035, **68251x faster**

Notes: Other experiments, including K-Means, node2vec, results on real quantum machine, etc., please refer to the paper

Conclusion: The calculation error brought by typical quantum mechanisms would incur in general little influence on the final numerical results given sufficient qubits. However, certain tasks e.g. ranking in K-Means could be more sensitive to quantum noise.